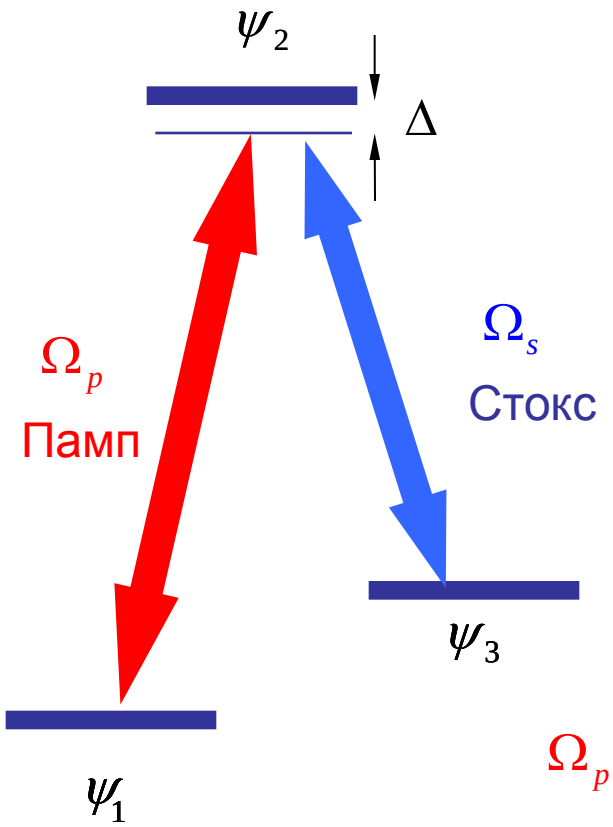
Нека да разгледаме система с три нива в ламбда конфигурация и нека да разложим вълновата функция в диабатен базис

$$\psi(t) = c_1(t)\psi_1 + c_2(t)\psi_2 + c_3(t)\psi_3$$

$\psi_1, \psi_2, \psi_3 -$ диабатен базис



Хамилтонияна в приближение на въртящата се вълна е

$$H(t) = \frac{1}{2}\begin{bmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 2\Delta(t) & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{bmatrix}$$

$\Omega_p(t), \Omega_s(t)$ Памп, Стокс

Раби честотите

$$\Omega_p(t) = -\mathbf{d}_{12}.\mathbf{E}_p(t)/\hbar$$

$$\Omega_s(t) = -\mathbf{d}_{23}.\mathbf{E}_s(t)/\hbar$$

$\Delta(t)$ е еднофотонният детунинг

Тоест уравнение на Шрьодингер в диабатен базис е

$$i\frac{d}{dt}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_p(t) & 0 \\ \frac{1}{2}\Omega_p(t) & \Delta(t) & \frac{1}{2}\Omega_s(t) \\ 0 & \frac{1}{2}\Omega_s(t) & 0 \end{bmatrix}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix}$$

$\phi_-(t), \phi_0(t), \phi_+(t) -$ Адиабатен базис (собствени вектори на H(t))

$$\psi(t) = a_-(t)\phi_-(t) + a_0(t)\phi_0(t) + a_+(t)\phi_+(t)$$

Връзката между адиабатен и диабатен базис е

$$a_-(t) = c_1 \sin\vartheta(t)\cos\varphi(t) - c_2 \sin\varphi(t) + c_3 \cos\varphi(t)\cos\vartheta(t)$$

$$a_0(t) = c_1 \cos\vartheta(t) - c_3 \sin\vartheta(t)$$ - тъмно състояние

$$a_+(t) = c_1 \sin\vartheta(t)\sin\varphi(t) + c_2 \cos\varphi(t) + c_3 \sin\varphi(t)\cos\vartheta(t)$$

$$\tan 2\varphi(t) = \frac{\Omega(t)}{\Delta(t)} \qquad \tan\vartheta(t) = \frac{\Omega_p(t)}{\Omega_s(t)} \qquad \Omega(t) = \sqrt{\Omega_p^2(t) + \Omega_s^2(t)}$$

уравнение на Шрьодингер в диабатен базис

$$i\frac{d}{dt}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_p(t) & 0 \\ \frac{1}{2}\Omega_p(t) & \Delta(t) & \frac{1}{2}\Omega_s(t) \\ 0 & \frac{1}{2}\Omega_s(t) & 0 \end{bmatrix}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix}$$

уравнение на Шрьодингер в адиабатен базис

$$i\frac{d}{dt}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega\cot\phi & i\dot{\theta}\sin\phi & i\dot{\phi} \\ -i\dot{\theta}\sin\phi & 0 & -i\dot{\theta}\cos\phi \\ -i\dot{\phi} & i\dot{\theta}\cos\phi & -\frac{1}{2}\Omega\tan\phi \end{bmatrix}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix}$$

Условия за адиабатна еволюция

$$\left|\dot{\theta}\sin\phi\right| << \tfrac{1}{2}\Omega\cot\phi \qquad \left|\dot{\theta}\cos\phi\right| << \tfrac{1}{2}\Omega\tan\phi \qquad \left|\dot{\phi}\right| << \tfrac{1}{2}\Omega\left|\cot\phi + \tan\phi\right|$$

уравнение на Шрьодингер в диабатен базис

$$i\frac{d}{dt}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\Omega_p(t) & 0 \\ \frac{1}{2}\Omega_p(t) & \Delta(t) & \frac{1}{2}\Omega_s(t) \\ 0 & \frac{1}{2}\Omega_s(t) & 0 \end{bmatrix}\begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix}$$

уравнение на Шрьодингер в адиабатен базис

$$i\frac{d}{dt}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega\cot\phi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\Omega\tan\phi \end{bmatrix}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix}$$

Условия за адиабатна еволюция

$$\left|\dot{\theta}\sin\phi\right| << \tfrac{1}{2}\Omega\cot\phi \qquad \left|\dot{\theta}\cos\phi\right| << \tfrac{1}{2}\Omega\tan\phi \qquad \left|\dot{\phi}\right| << \tfrac{1}{2}\Omega\left|\cot\phi + \tan\phi\right|$$

Адиабатна еволюция

$$i\frac{d}{dt}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega\cot\phi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\Omega\tan\phi \end{bmatrix}\begin{bmatrix} a_+ \\ a_0 \\ a_- \end{bmatrix}$$

Условия за адиабатна еволюция

$$\left|\dot{\theta}\sin\phi\right| << \tfrac{1}{2}\Omega\cot\phi \qquad \left|\dot{\theta}\cos\phi\right| << \tfrac{1}{2}\Omega\tan\phi \qquad \left|\dot{\phi}\right| << \tfrac{1}{2}\Omega\left|\cot\phi + \tan\phi\right|$$

Ако имаме адиабатна еволюция и първоначално сме само в тъмното състояние, то системата е винаги в тъмното състояние

$$a_0(t) = c_1\cos\vartheta(t) - c_3\sin\vartheta(t)$$ - тъмно състояние

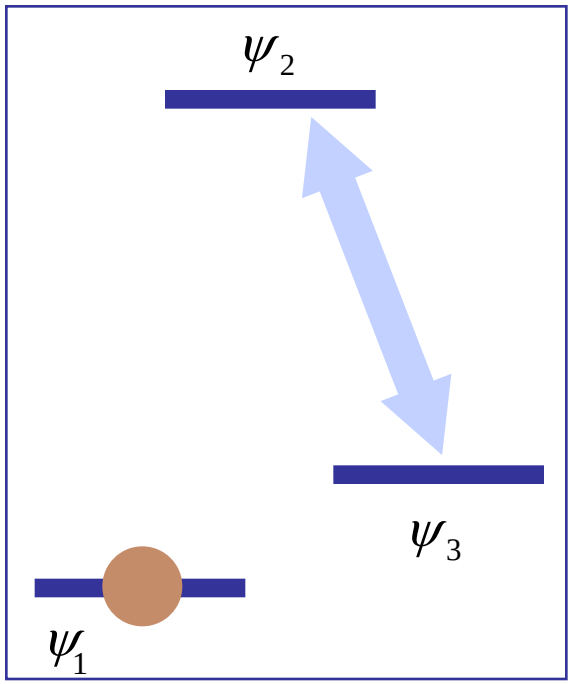
Пълно прехвърляне на заселеността от основно състояние ψ_1 в крайно ψ_3
без да се заселва междинното състояние ψ_2

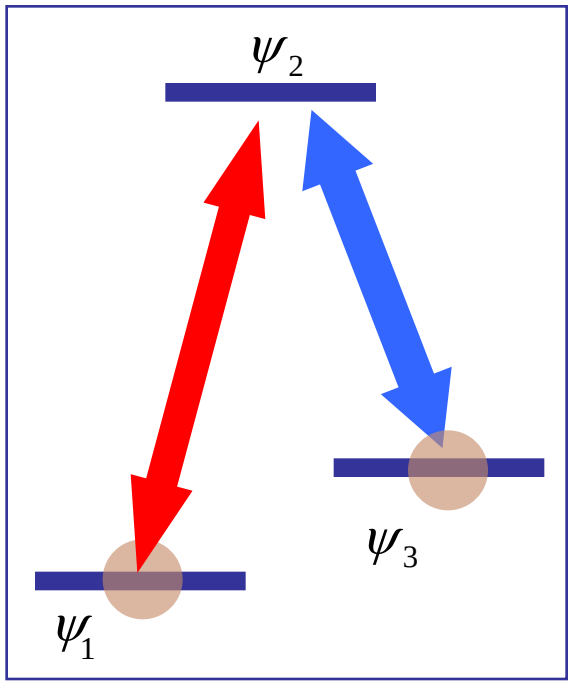
Условия: Адиабатини условия и контраинтуитивна наредба на импулсите

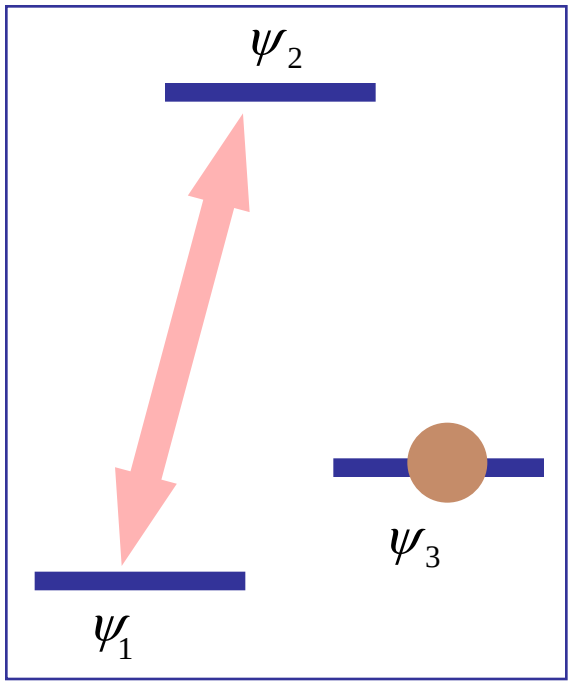
Резултати: Висока ефективност на прехвърляне на заселсността

Обяснение: $a_0(t) = c_1 \cos\vartheta(t) - c_3 \sin\vartheta(t)$ - Тъмно състояние

$$\tan\vartheta(t) = \Omega_p(t)/\Omega_s(t)$$







$\vartheta(-\infty) = 0$

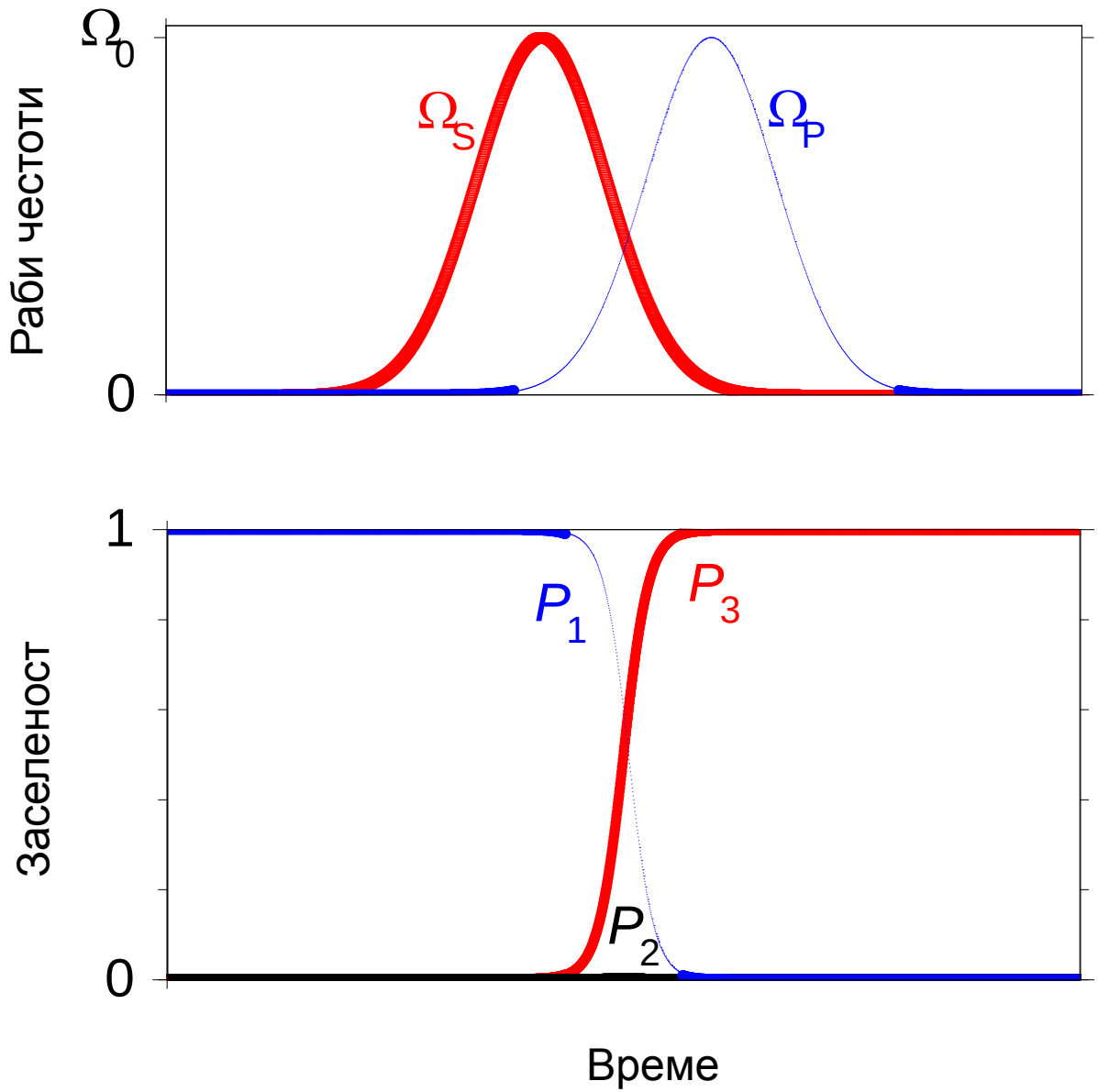
$a_0(-\infty) = c_1$

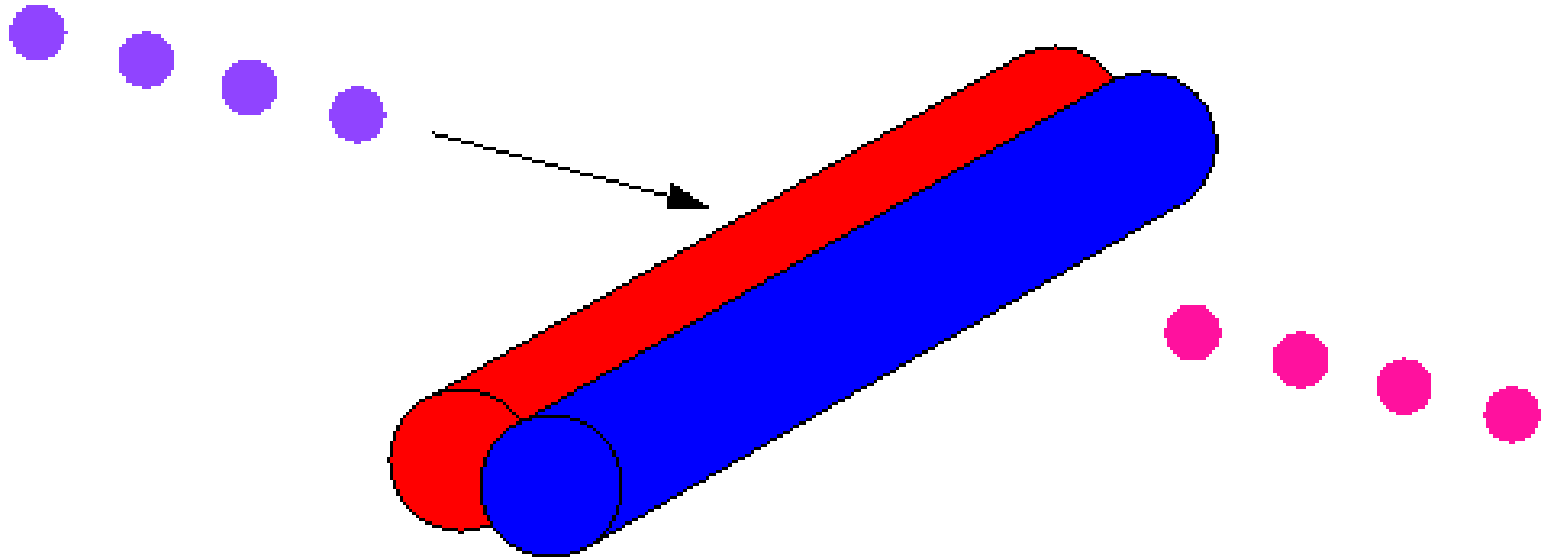
Тъмно състояние

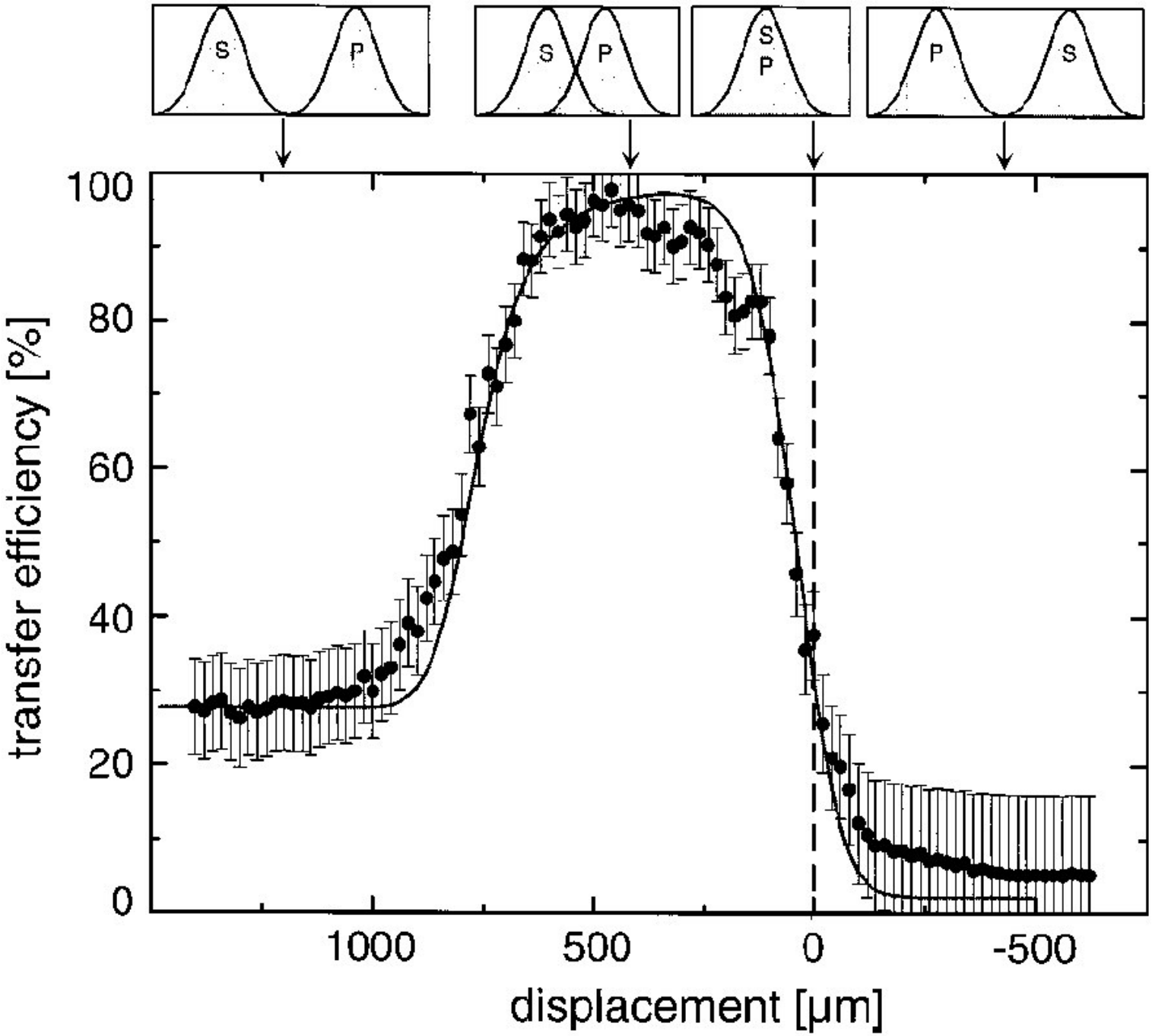
$\vartheta(\infty) = \pi/2$

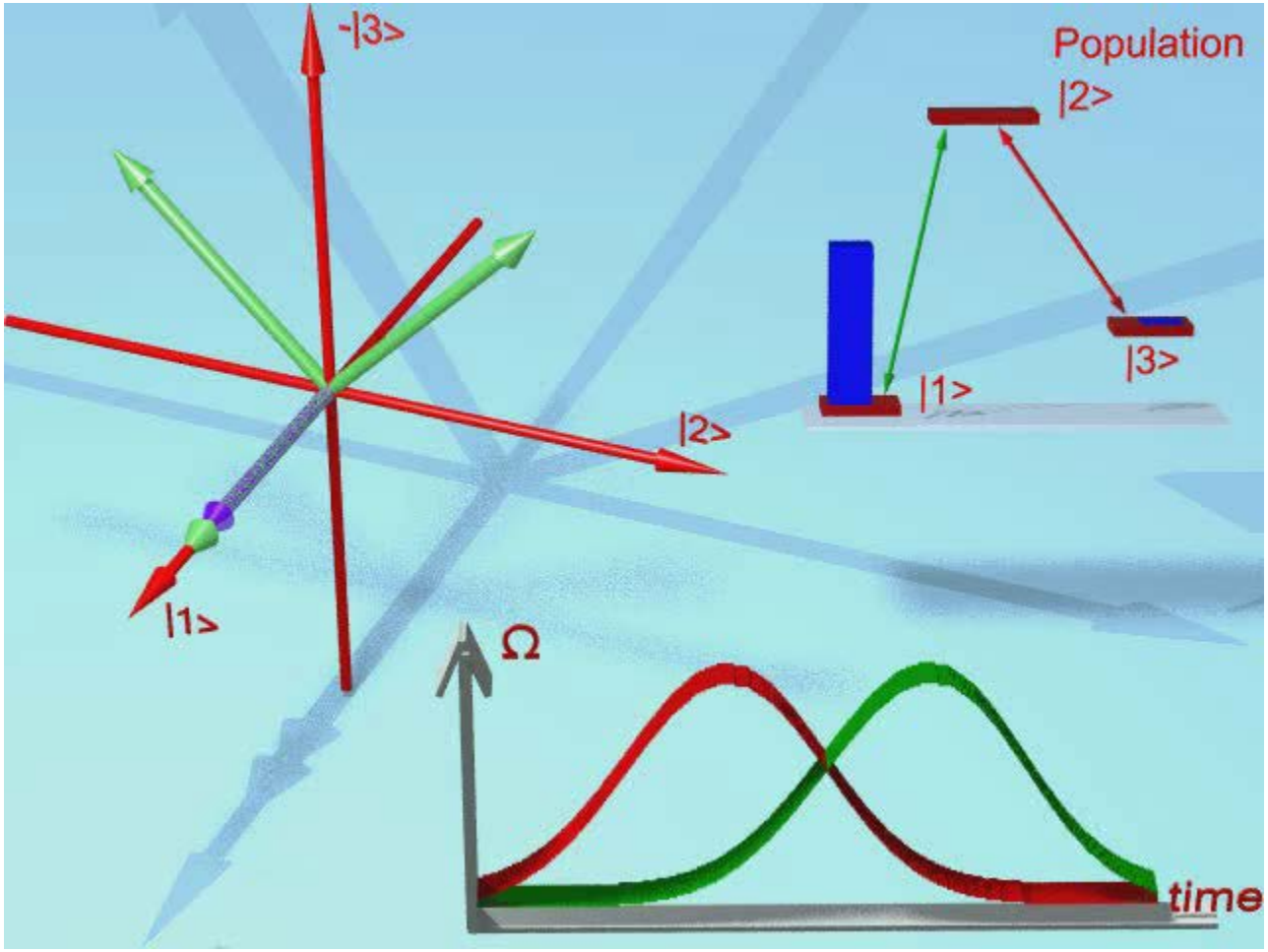
$a_0(\infty) = -c_3$

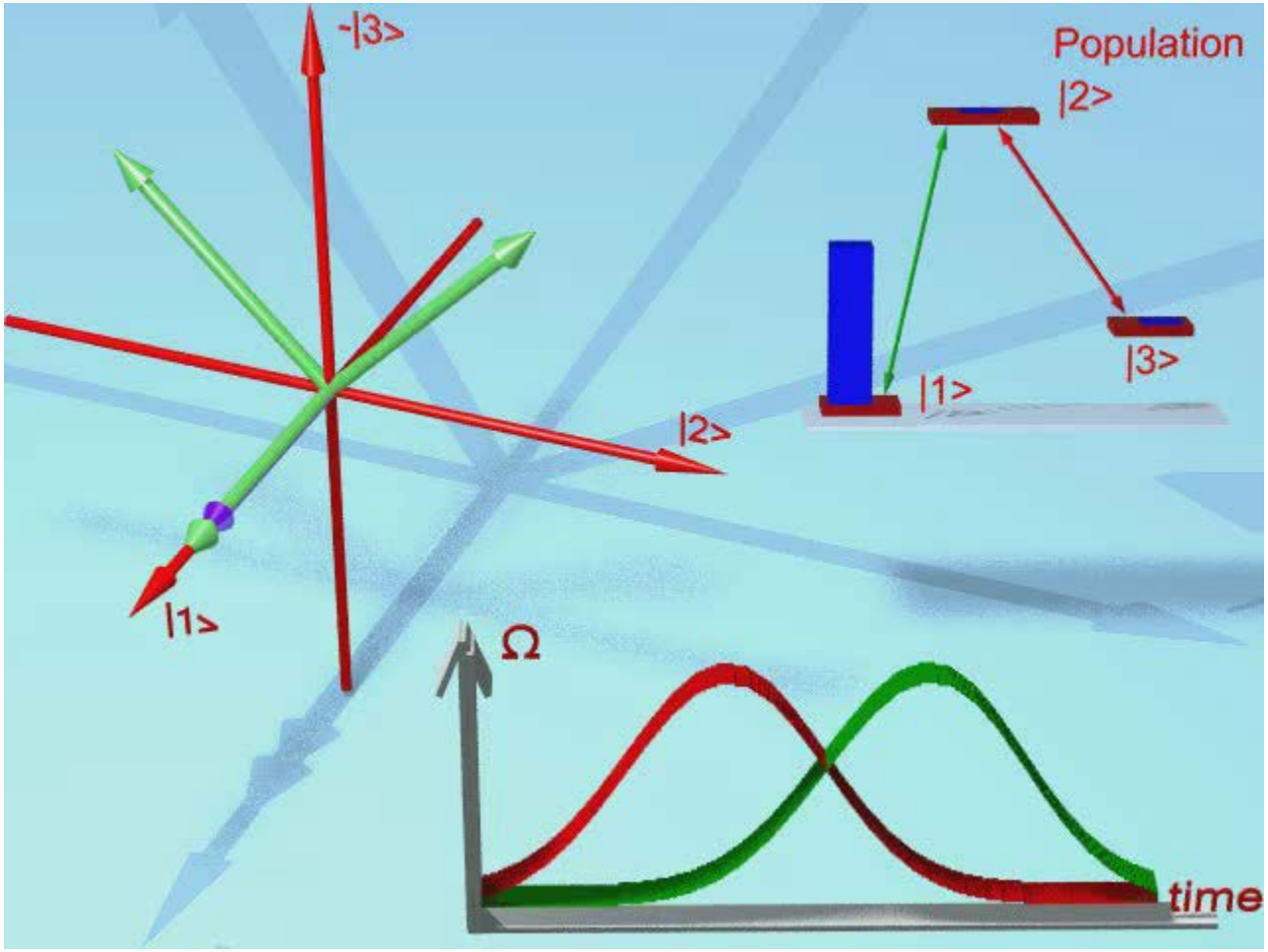
Тъмно състояние


Раби честоти
Ω_0
0
Ω_S
Ω_P
Заселеност
1
0
P_1
P_3
P_2
Време




S
P
S
P
S
P
P
S
100
80
60
40
20
0
transfer efficiency [%]
1000
500
0
-500
displacement [µm]


-|3>
|2>
|1>
Population
|2>
|3>
|1>
Ω
time


-|3>
|2>
|1>
Population
|2>
|3>
|1>
Ω
time

Предимства на СТИРАП

- Изцяло прехвърляне на заселеност между ψ_1 и ψ_3
- Не заселва състояние ψ_2

(двумерно Хилбертово пространство)

- Устойчива техника : Не е чувствителна към формата и големината на импулсите

- не изисква точна площ на импулсите

При поглъщане на фотон освен енергия атома придобива и импулс $p = \hbar k$

При поглъщане на фотон освен енергия атома придобива и импулс $p = \hbar k$

При излъчване на фотон атома отново придобива импулс $p = \hbar k$

При поглъщане на фотон освен енергия атома придобива и импулс $p = \hbar k$

При излъчване на фотон атома отново придобива импулс $p = \hbar k$

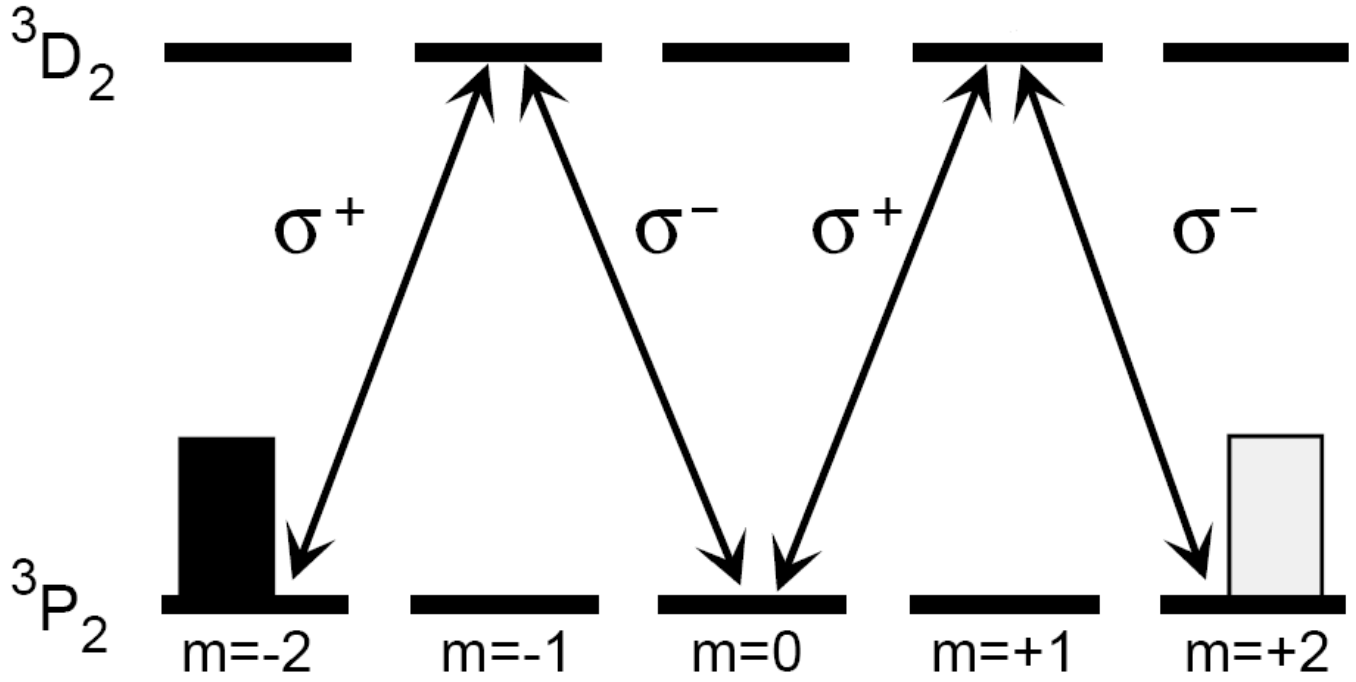
Атомен разделите, Атомен интерферометър

При поглъщане на фотон освен енергия атома придобива и импулс $p = \hbar k$

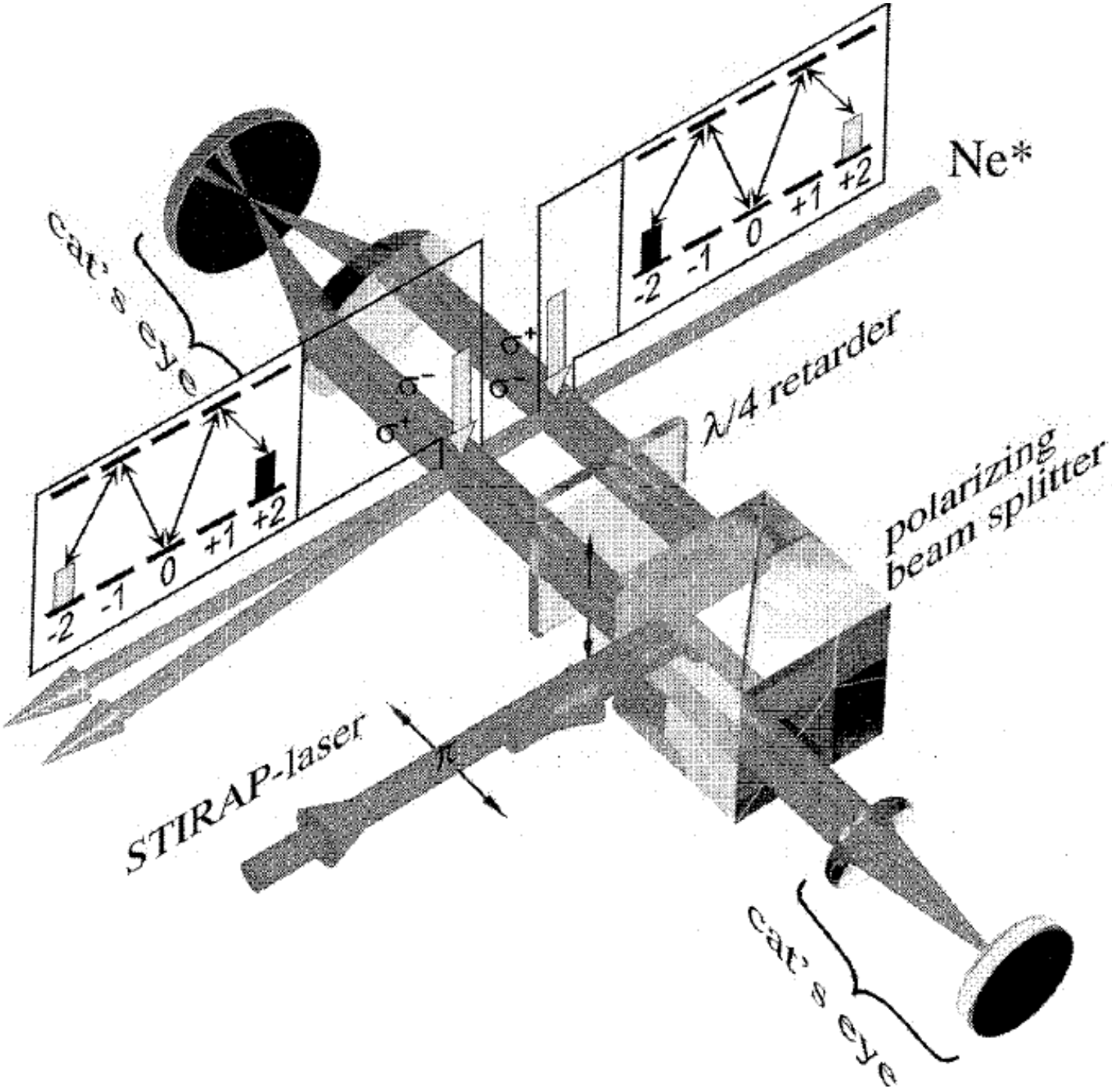
При излъчване на фотон атома отново придобива импулс $p = \hbar k$

Атомен разделите, Атомен интерферометър

Атоми на Ne



Общ придобит импулс $\Delta p = 4\hbar k$


Ne*
cat's eye
λ/4 retarder
polarizing
beam splitter
STIRAP-laser
π
σ+
σ−
σ−
σ+
-2
-1
0
+1
+2
-2
-1
0
+1
+2
cat's eye

Предимства пред атомен интерферометър с две нива:

Предимства пред атомен интерферометър с две нива:

1) Голяма ефективност при преноса на заселеност понеже не се изисква точна площ на импулса и резонанс; $\Rightarrow$ всички атоми придобиват еднакъв импулс

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2) Стимулиран преход $\Rightarrow$ нямаме случайно излъчване и произволна посока на придобитият импулс

Предимства пред атомен интерферометър с две нива:

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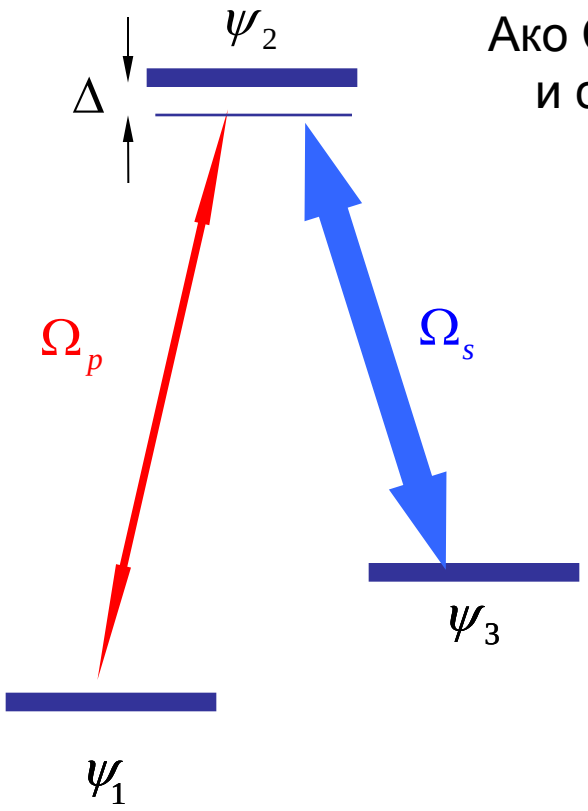
2) Стимулиран преход $\Rightarrow$ нямаме случайно излъчване и произволна посока на придобитият импулс

3) Придобиване на 4 пъти по-голям импулс сравнение с атом с две нива

Електромагнитна индуцирана прозрачност (ЕИП)



Ако Стокс полето е силно, то системата е в тъмно състояние и следователно не заселва състояние ψ_2 тоест средата става прозрачна

$$\left|darkstate\right\rangle = \frac{\Omega_S \left|\psi_1\right\rangle - \Omega_P \left|\psi_3\right\rangle}{\sqrt{\Omega_S^2 + \Omega_P^2}}$$

Частичен СТИРАП

$$a_0(t) = c_1 \cos \vartheta(t) - c_3 \sin \vartheta(t)$$ - тъмно състояние

Частичен СТИРАП

$$a_0(t) = c_1 \cos \vartheta(t) - c_3 \sin \vartheta(t)$$ - тъмно състояние

$$\tan \vartheta(\infty) = \frac{\Omega_p(\infty)}{\Omega_s(\infty)}$$

Частичен СТИРАП

$$a_0(t) = c_1 \cos \vartheta(t) - c_3 \sin \vartheta(t)$$ - тъмно състояние

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$$\vartheta(-\infty) = 0$$

$$a_0(-\infty) = c_1$$

Частичен СТИРАП

$a_0(t) = c_1 \cos\vartheta(t) - c_3 \sin\vartheta(t)$ - тъмно състояние

$$\tan\vartheta(\infty) = \frac{\Omega_p(\infty)}{\Omega_s(\infty)}$$

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$$\vartheta(\infty) = \alpha$$

$$a_0(\infty) = c_1 \cos\alpha - c_3 \sin\alpha$$

Частичен СТИРАП

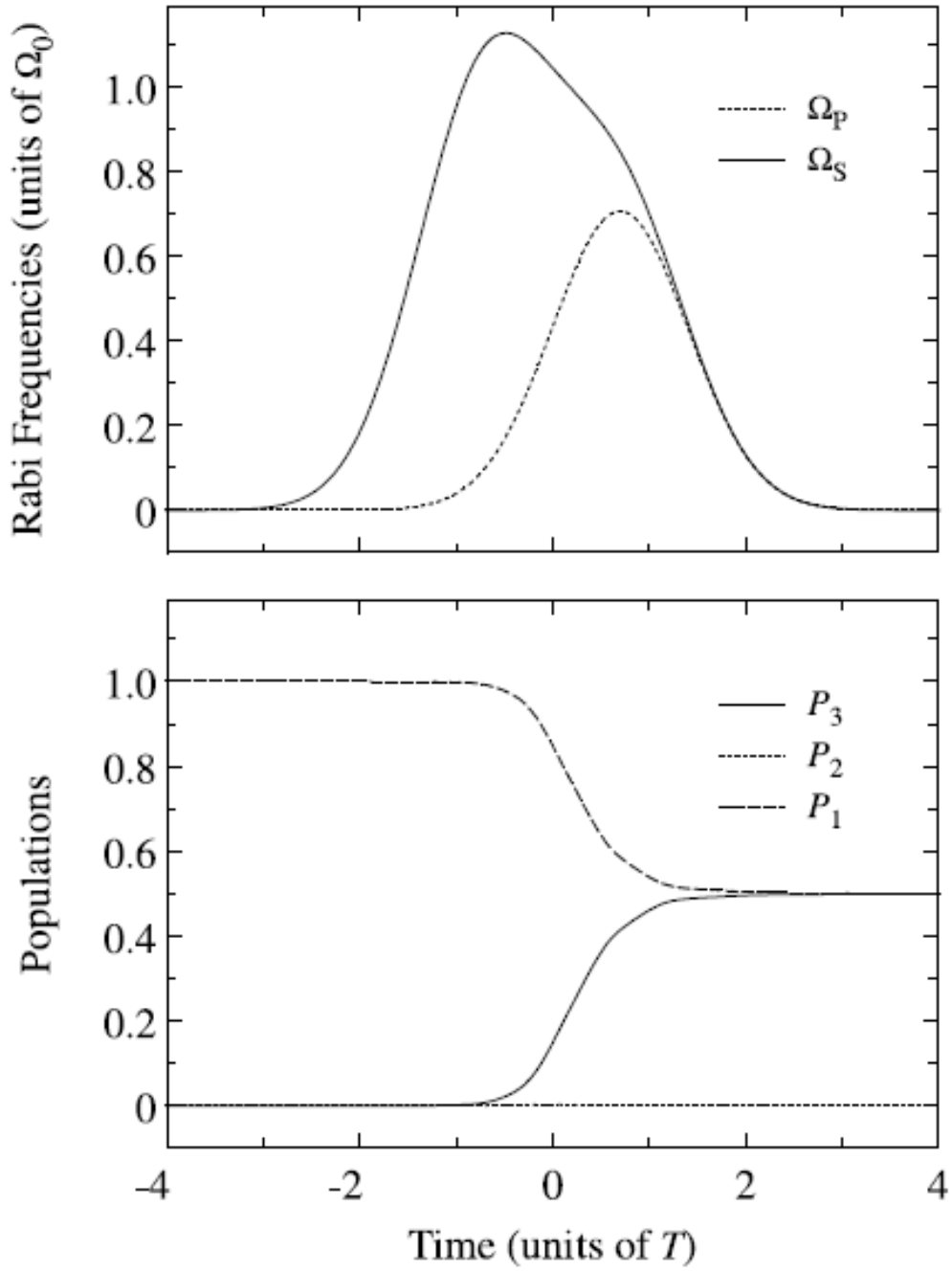
$a_0(t) = c_1 \cos \vartheta(t) - c_3 \sin \vartheta(t)$ - тъмно състояние

$$\tan \vartheta(\infty) = \frac{\Omega_p(\infty)}{\Omega_s(\infty)}$$

$$\vartheta(-\infty) = 0$$
$$a_0(-\infty) = c_1$$

$$\vartheta(\infty) = \alpha$$
$$a_0(\infty) = c_1 \cos\alpha - c_3 \sin\alpha$$

$$\Omega_P(t) = \Omega_0 \sin\alpha \, e^{-(t-\tau)^2/T^2}$$
$$\Omega_S(t) = \Omega_0 e^{-(t+\tau)^2/T^2} + \Omega_0 \cos\alpha \, e^{-(t-\tau)^2/T^2}.$$


Rabi Frequencies (units of Ω_0)
1.0
0.8
0.6
0.4
0.2
0
Ω_P
Ω_S
Populations
1.0
0.8
0.6
0.4
0.2
0
P_3
P_2
P_1
-4
-2
0
2
4
Time (units of T)

уравнение на Шрьодингер за резонансна Λ система

сменяме променливите

$$i\frac{d}{dt}\begin{bmatrix}\mathbf{c}_1(t)\\ \mathbf{c}_2(t)\\ \mathbf{c}_3(t)\end{bmatrix}=\begin{bmatrix}0 & \Omega_p(t) & 0\\ \Omega_p(t) & 0 & \Omega_s(t)\\ 0 & \Omega_s(t) & 0\end{bmatrix}\begin{bmatrix}\mathbf{c}_1(t)\\ \mathbf{c}_2(t)\\ \mathbf{c}_3(t)\end{bmatrix}$$

$$\mathbf{c}_1(t)=w(t),$$
$$\mathbf{c}_2(t)=iv(t),$$
$$\mathbf{c}_3(t)=-u(t)$$

уравнение на Шрьодингер преминава в уравнение на Блох

$$\frac{d}{dt}\begin{bmatrix}u(t)\\ v(t)\\ w(t)\end{bmatrix}=\begin{bmatrix}0 & -\Omega_s(t) & 0\\ \Omega_s(t) & 0 & -\Omega_P(t)\\ 0 & \Omega_P(t) & 0\end{bmatrix}\begin{bmatrix}u(t)\\ v(t)\\ w(t)\end{bmatrix}$$

Phys. Rev. A 73, 053402 (2006).

Други СТИРАП аналози

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languages

Frenet–Serret formulas

From Wikipedia, the free encyclopedia

"Binormal" redirects here. For the category-theoretic meaning of this word, see Normal morphism.

In vector calculus, the **Frenet–Serret formulas** describe the kinematic properties of a particle which moves along a continuous, differentiable curve in three-dimensional Euclidean space $\mathbf{R}^3$. More specifically, the formulas describe the derivatives of the so-called **tangent, normal, and binormal** unit vectors in terms of each other. The formulas are named after the two French mathematicians who independently discovered them: Jean Frédéric Frenet, in his thesis of 1847, and Joseph Alfred Serret in 1851. Vector notation and linear algebra currently used to write these formulas was not yet in use at the time of their discovery.

The tangent, normal, and binormal vectors, often called **T**, **N**, and **B**, or collectively the **Frenet–Serret frame** or **TNB frame** are defined as follows:

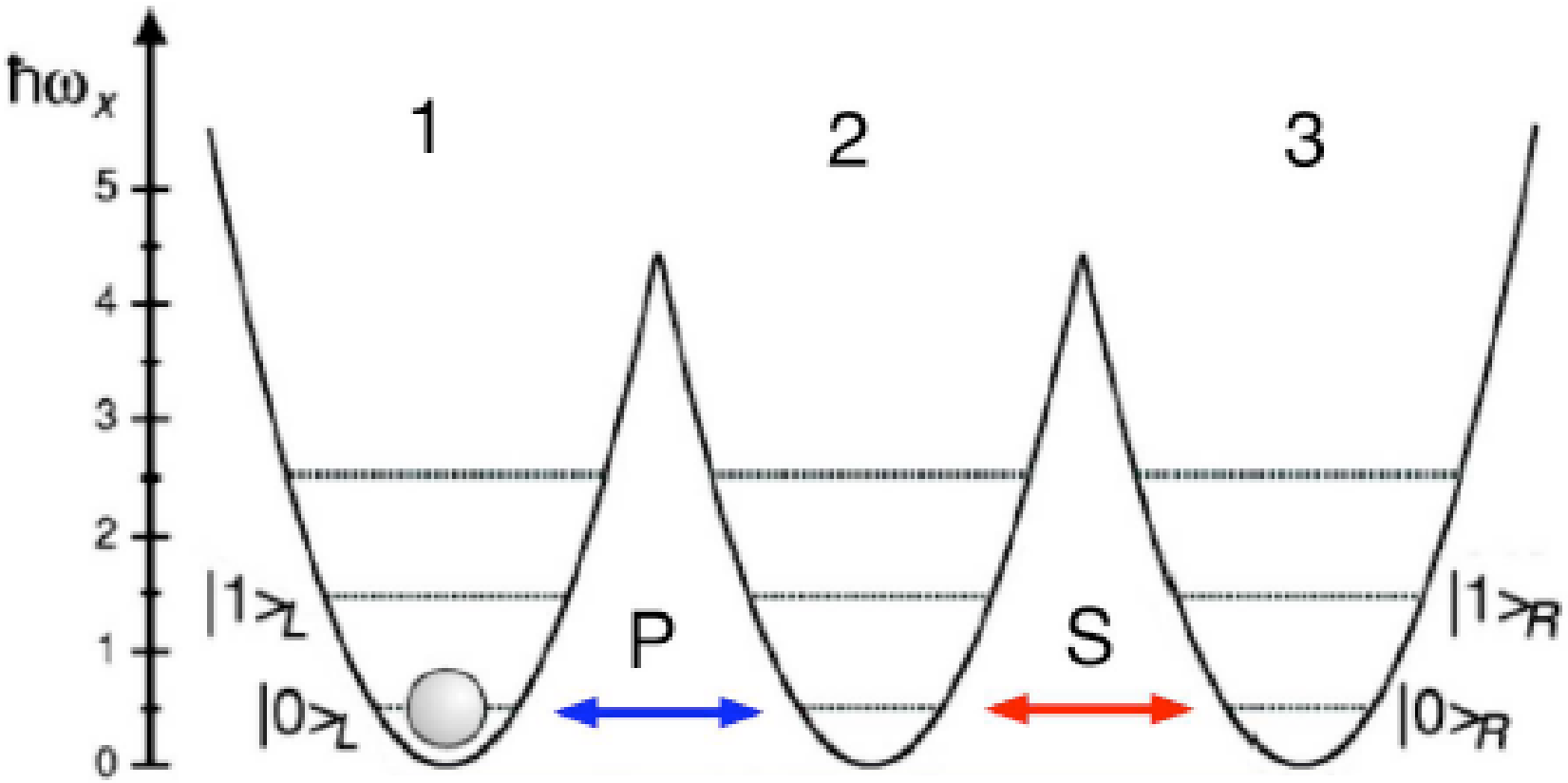
- **T** is the unit vector tangent to the curve, pointing in the direction of motion.
- **N** is the derivative of **T** with respect to the arclength parameter of the curve, divided by its length.
- **B** is the cross product of *T* and *N*.

The Frenet–Serret formulas are

$$\begin{aligned}
\frac{d\mathbf{T}}{ds} &= \kappa\mathbf{N} \\
\frac{d\mathbf{N}}{ds} &= -\kappa\mathbf{T} + \tau\mathbf{B} \\
\frac{d\mathbf{B}}{ds} &= -\tau\mathbf{N}
\end{aligned}$$

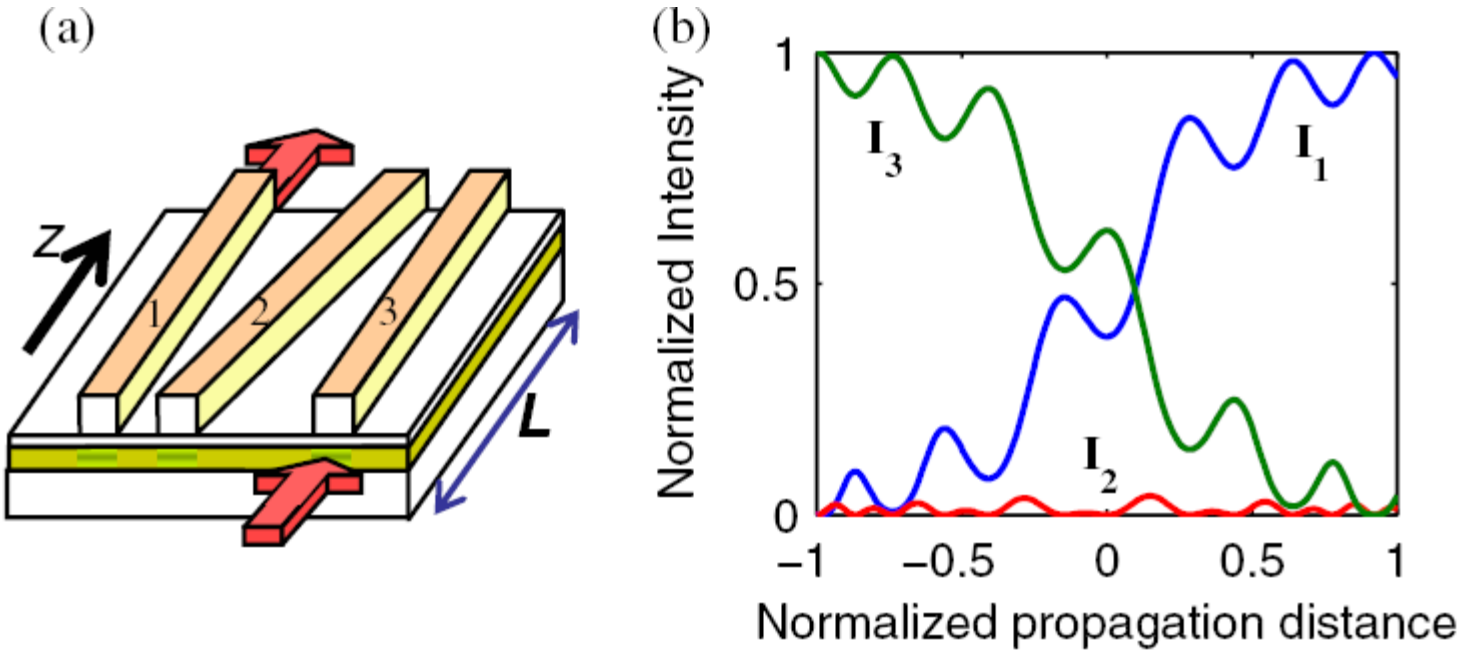
where *d*/*ds* is the derivative with respect to arclength, κ is the curvature and τ is the torsion of the curve. This formula effectively defines the curvature and torsion of a space curve.

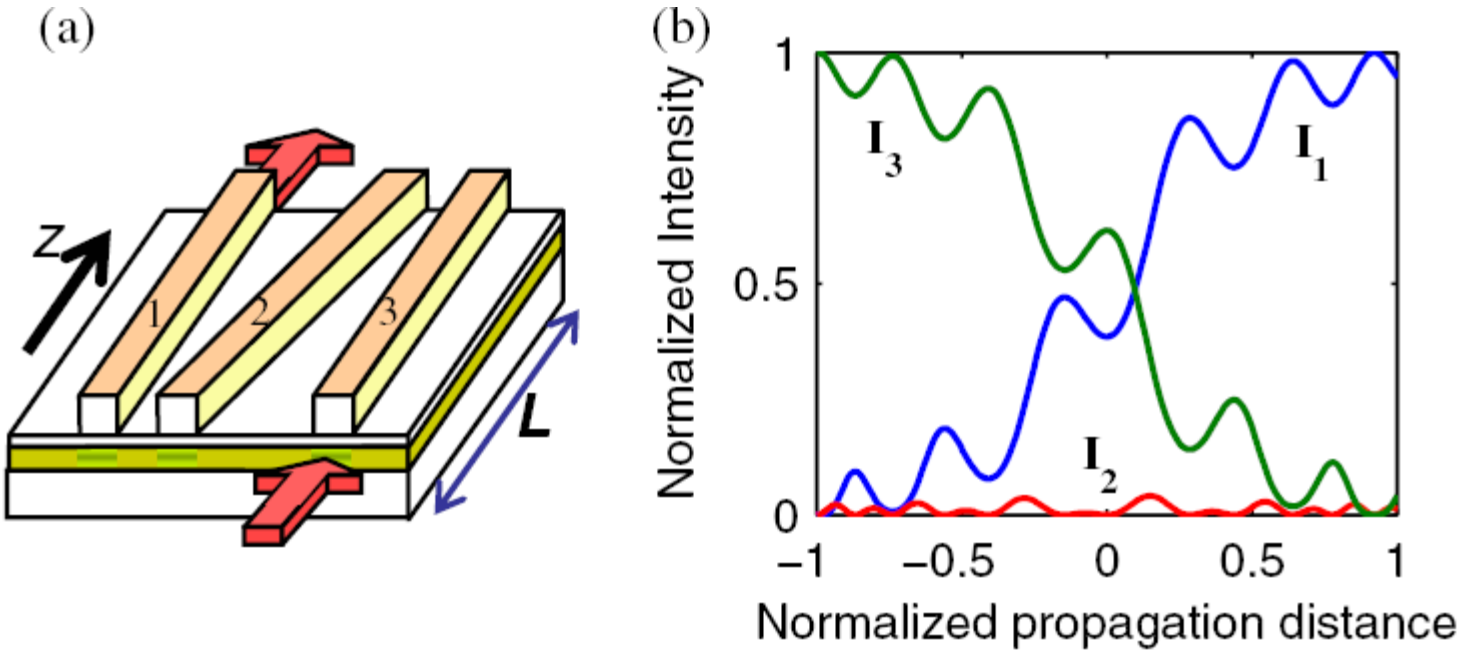
Contents [hide]


ħω_x
5
4
3
2
1
0
1
2
3
|1>_L
|0>_L
P
S
|1>_R
|0>_R

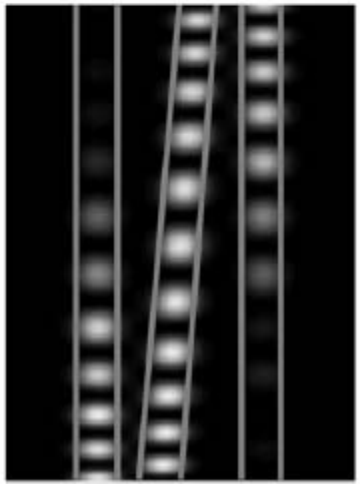
STIRAP in waveguides

Paspalakis, Optics Communications 258, 30 (2006)

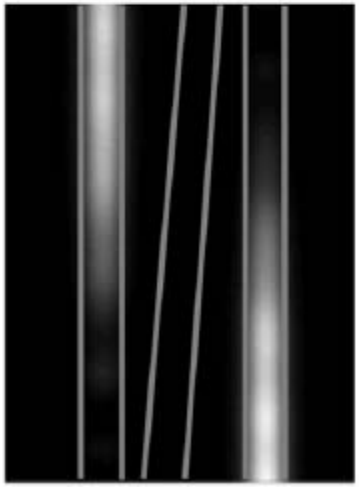




Silberberg et al. PRL 101, 193901 (2008)



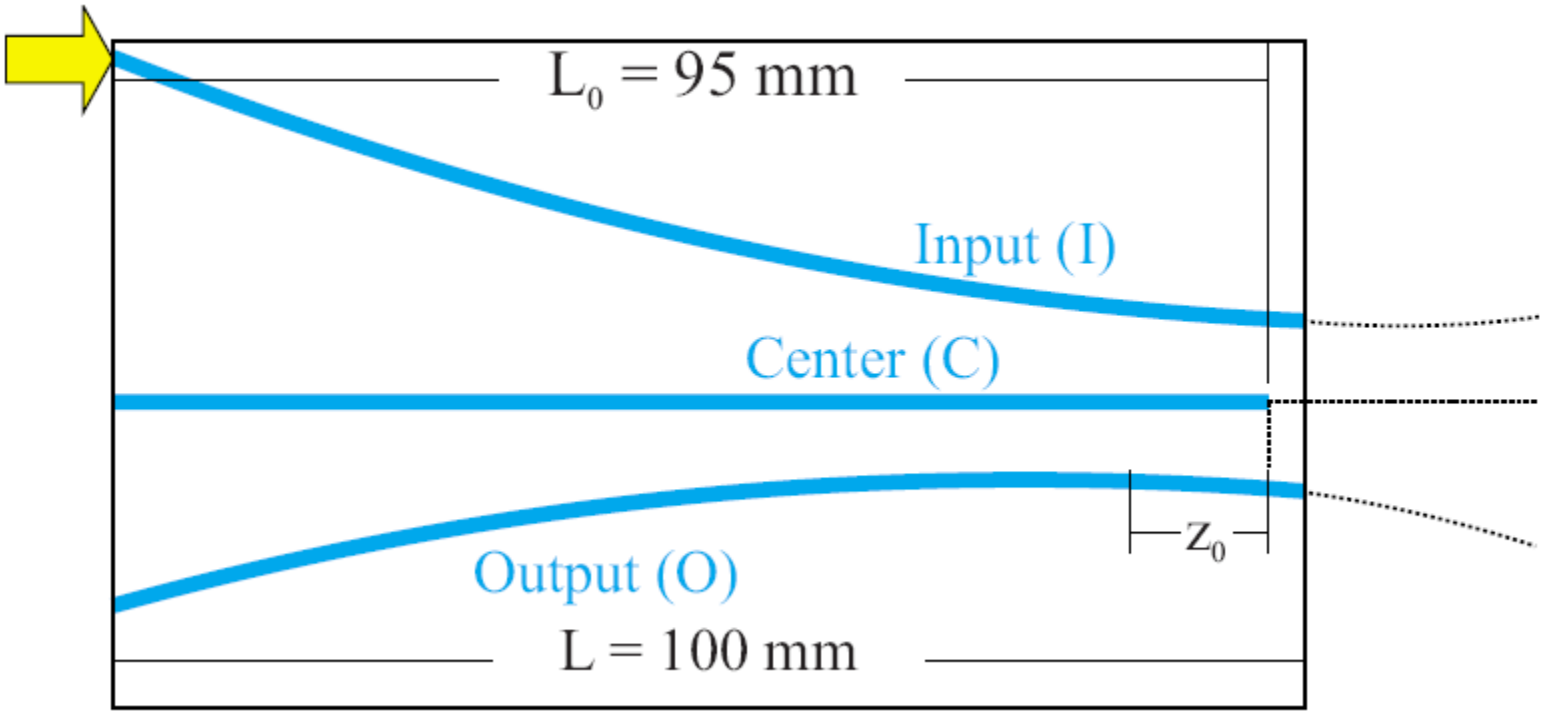
intuitive



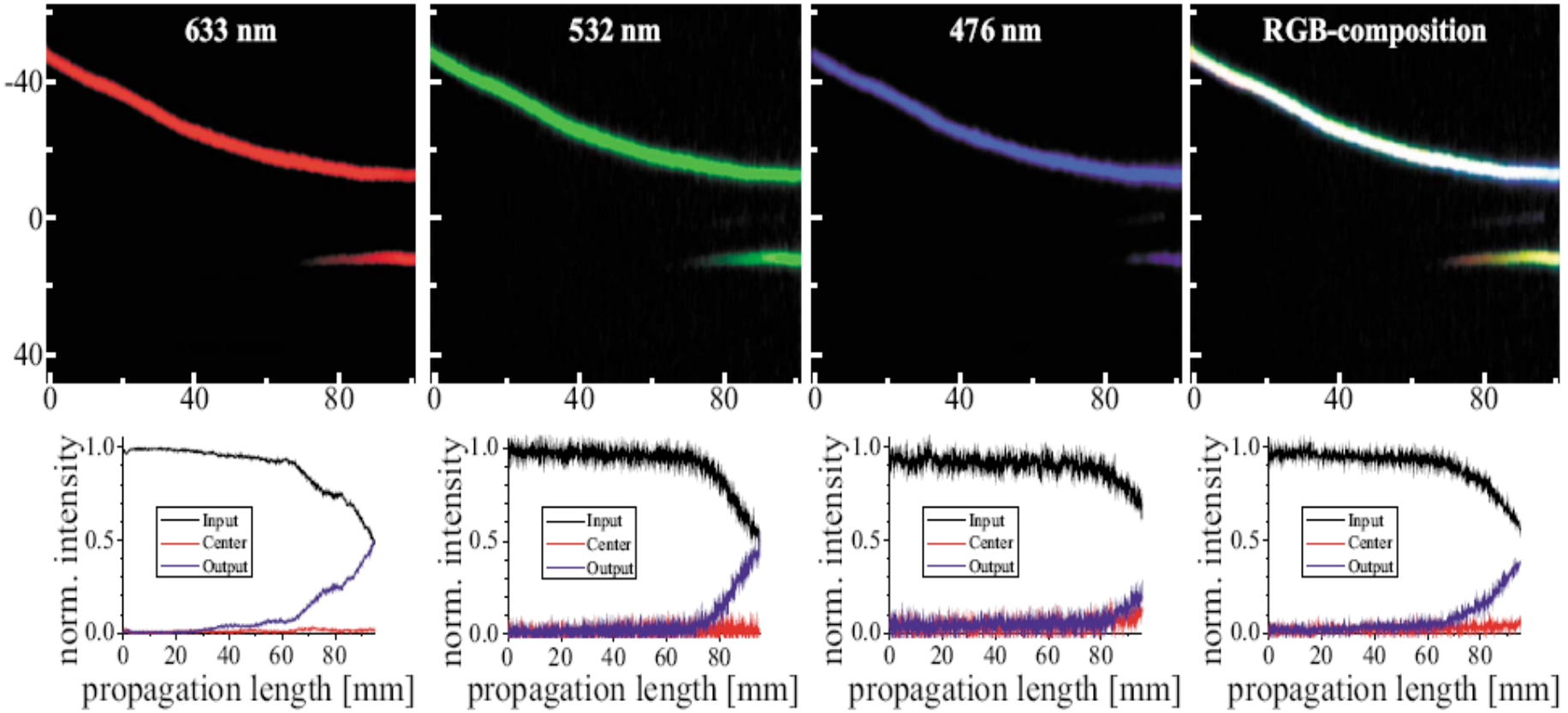
contraintuitive

Silberberg et al. PRL 101, 193901 (2008)

Fractional STIRAP in waveguides

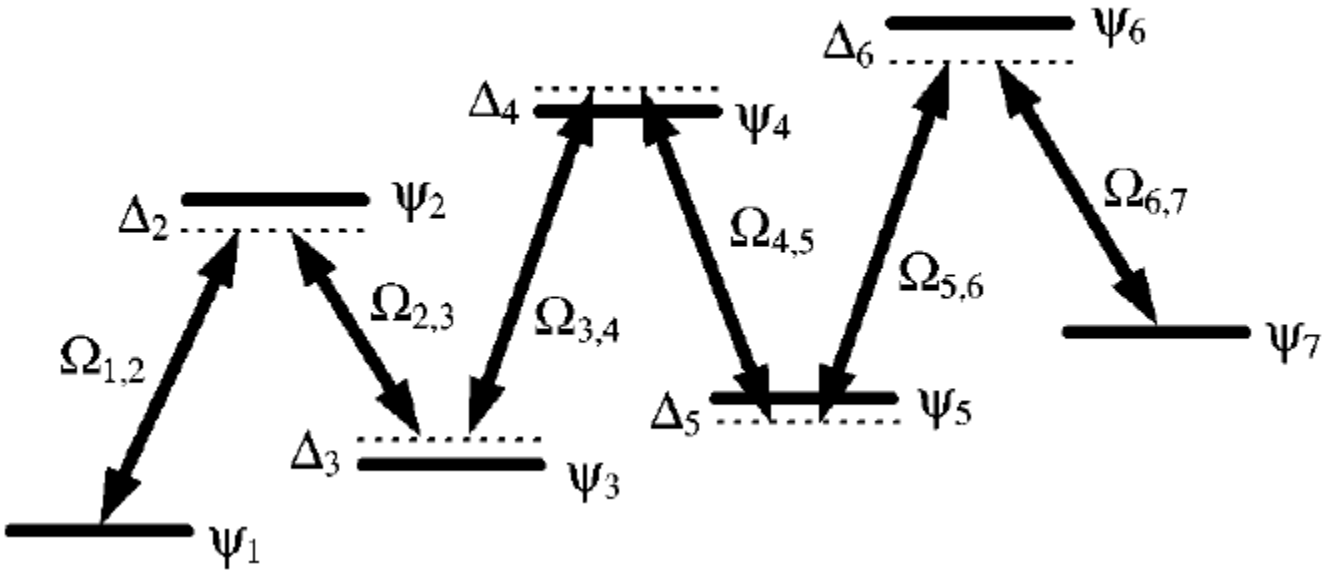


Longhi et al. Appl. Phys. Lett. 95, 261102 (2009)



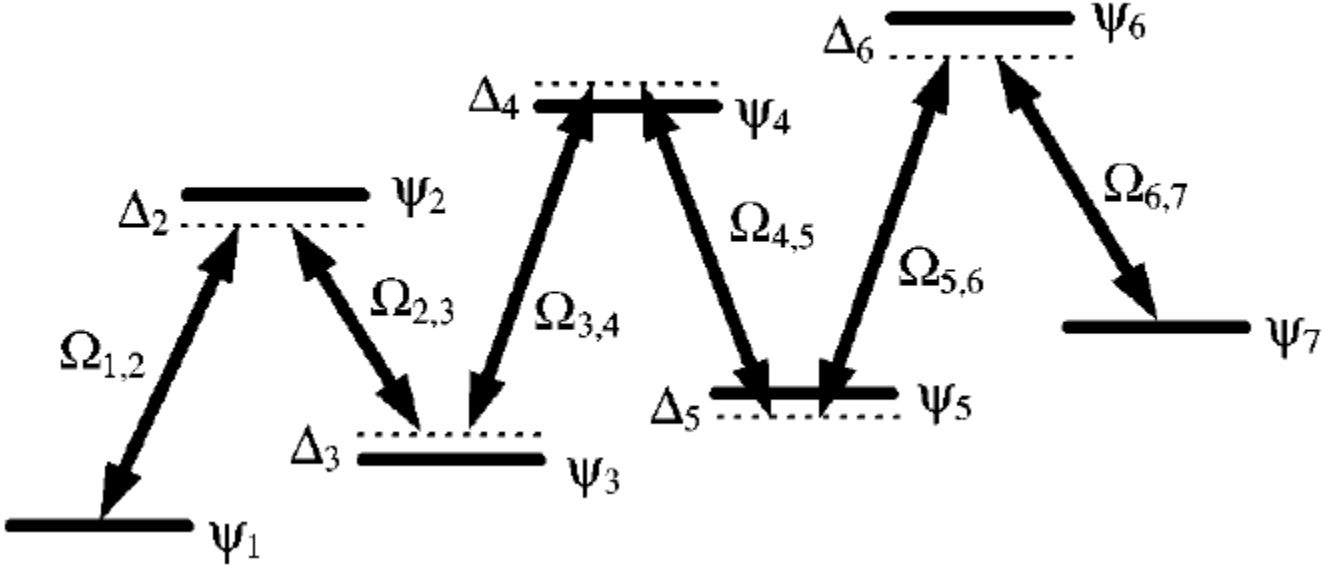
Longhi et al. Appl. Phys. Lett. 95, 261102 (2009)

Multiple STIRAP



"In the *on-resonance* case, an adiabatic-transfer state always exists for an odd number of states while it never exists for an even number of states"

Vitanov, PRA **58**, 2295 (1998)



$$\Delta_1 = \Delta_2 = ... = \Delta_N = 0$$

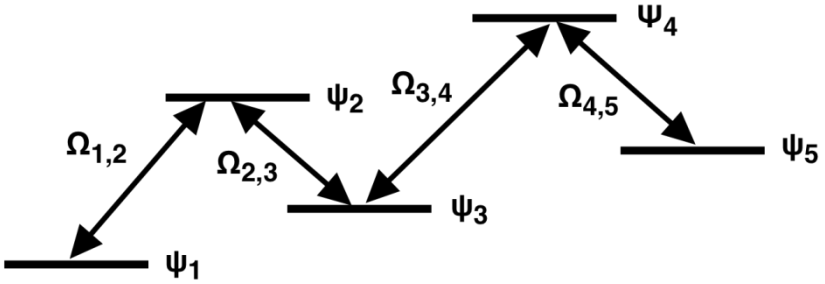
$$DarkState = (v_1, v_2, ...v_{N-1}, v_N)^T$$

$$v_{2k} = 0$$

$$v_{2k+1} = \frac{(-1)^k}{\mathrm{N}} \Omega_{1,2}\Omega_{2,3}...\Omega_{2k-1,2k}\Omega_{2k+2,2k+3}....\Omega_{N-1,N}$$

Vitanov, PRA **58**, 2295 (1998)

Quantum popul. dynamics vs. Waveguide optics



N states coupled by coherent laser fields in resonance

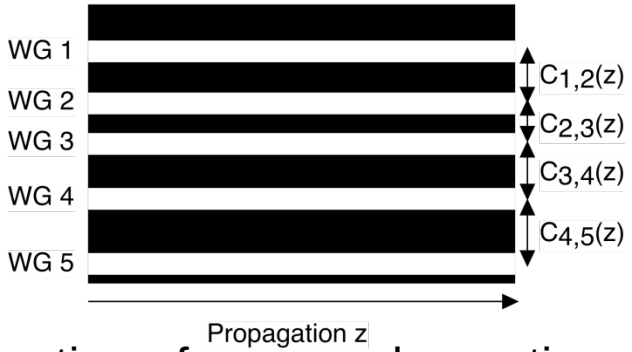
Schrödinger equation for the population amplitudes :

$$i\frac{d}{dt}A(t) = H(t)\cdot A(t)$$

A_n(t): Population amplitude in state *n*

$\Omega_{n,n+1}(t)$: Rabi frequencies between states *n* and *n+1*

$$H(t) = \begin{pmatrix} 0 & \Omega_{1,2}(t) & 0 & 0 & 0 \\ \Omega_{1,2}(t) & 0 & \Omega_{2,3}(t) & 0 & 0 \\ 0 & \Omega_{2,3}(t) & 0 & \Omega_{3,4}(t) & 0 \\ 0 & 0 & \Omega_{3,4}(t) & 0 & \Omega_{4,5}(t) \\ 0 & 0 & 0 & \Omega_{4,5}(t) & 0 \end{pmatrix} \quad A(t) = \begin{bmatrix} A_1(t) \\ A_2(t) \\ A_3(t) \\ A_4(t) \\ A_5(t) \end{bmatrix}$$



Propagation of a monochromatic wave in a waveguide array.

Coupled mode theory (coupling between nearest neighbors) :

$$i\frac{d}{dz}A(z) = H(z)\cdot A(z)$$

$A_n(z)$: Mode amplitude in waveguide *n*

$C_{n,n+1}(z)$: Coupling constant between waveguides *n* and *n+1*

$$H(z) = \begin{pmatrix} 0 & C_{1,2}(z) & 0 & 0 & 0 \\ C_{1,2}(z) & 0 & C_{2,3}(z) & 0 & 0 \\ 0 & C_{2,3}(z) & 0 & C_{3,4}(z) & 0 \\ 0 & 0 & C_{3,4}(z) & 0 & C_{4,5}(z) \\ 0 & 0 & 0 & C_{4,5}(z) & 0 \end{pmatrix} \quad A(z) = \begin{bmatrix} A_1(z) \\ A_2(z) \\ A_3(z) \\ A_4(z) \\ A_5(z) \end{bmatrix}$$

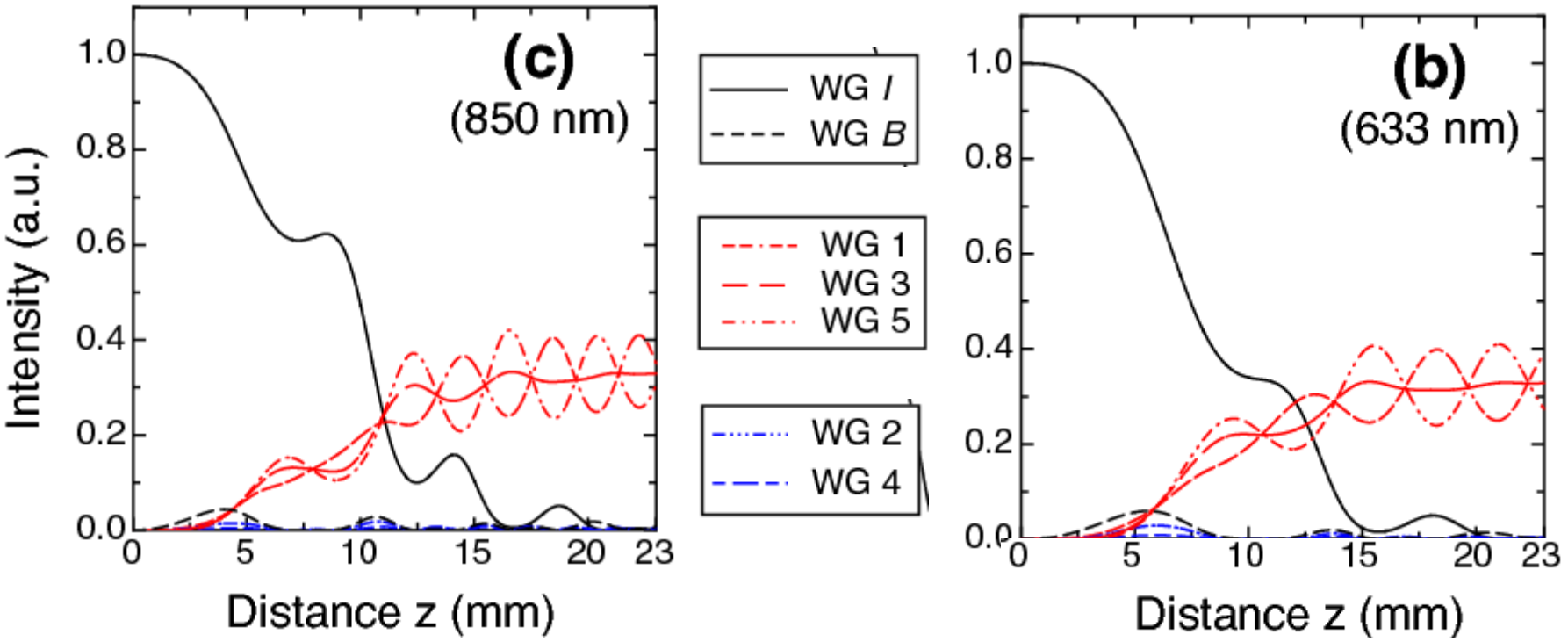
Equivalence between the temporal dimension in quantum physics and the spatial dimension in optics

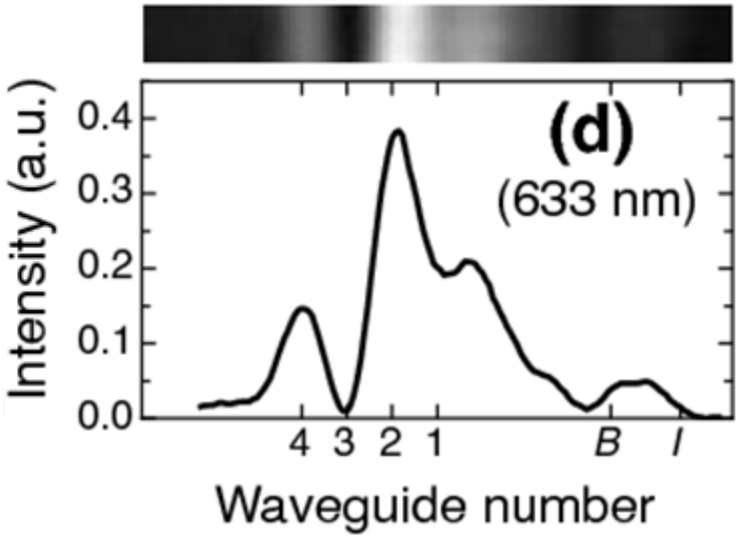
Planar N-fold achromatic beam splitter

$$DarkState(z=0)=(1,0,0,0,0,0,0)^T$$

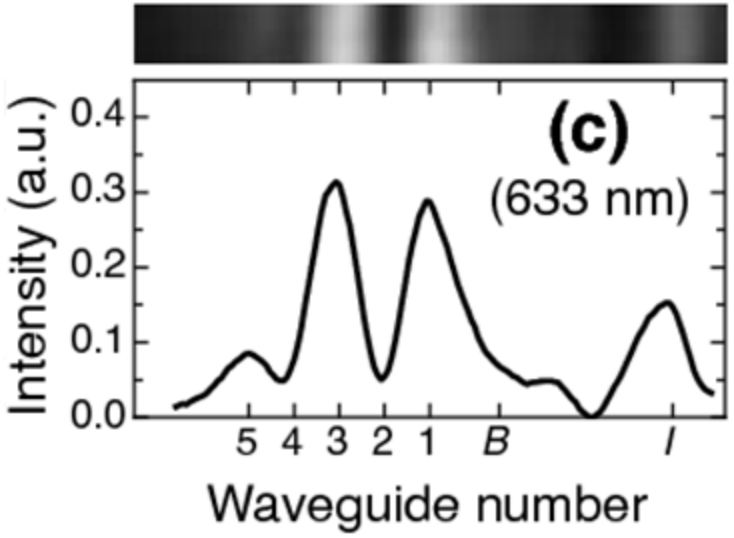
$$DarkState(z=L)=(0,0,-1,0,1,0,-1)^T$$

C. Ciret, V. Coda, A. Rangelov, D. Neshev, and G. Montemezzani,
“Planar achromatic multiple beam splitter by adiabatic light transfer”

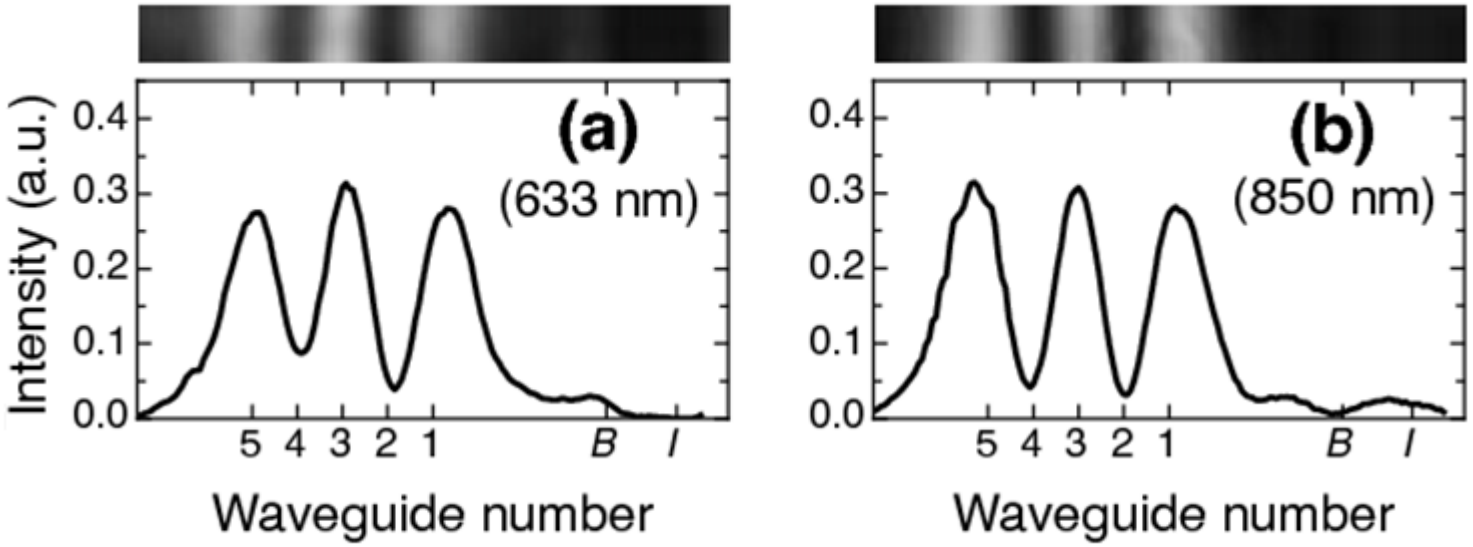

(c)
(850 nm)
Intensity (a.u.)
Distance z (mm)
WG I
WG B
WG 1
WG 3
WG 5
WG 2
WG 4
(b)
(633 nm)
Distance z (mm)



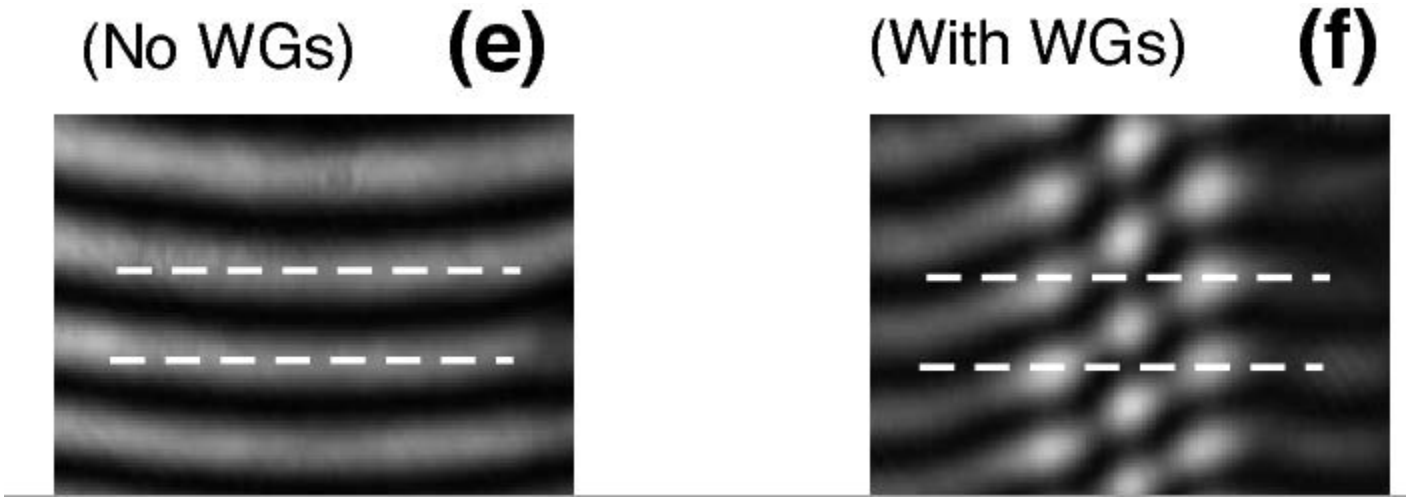
When N=4 (even numbers of waveguides in the array) there is no adiabatic transfer state (dark state)



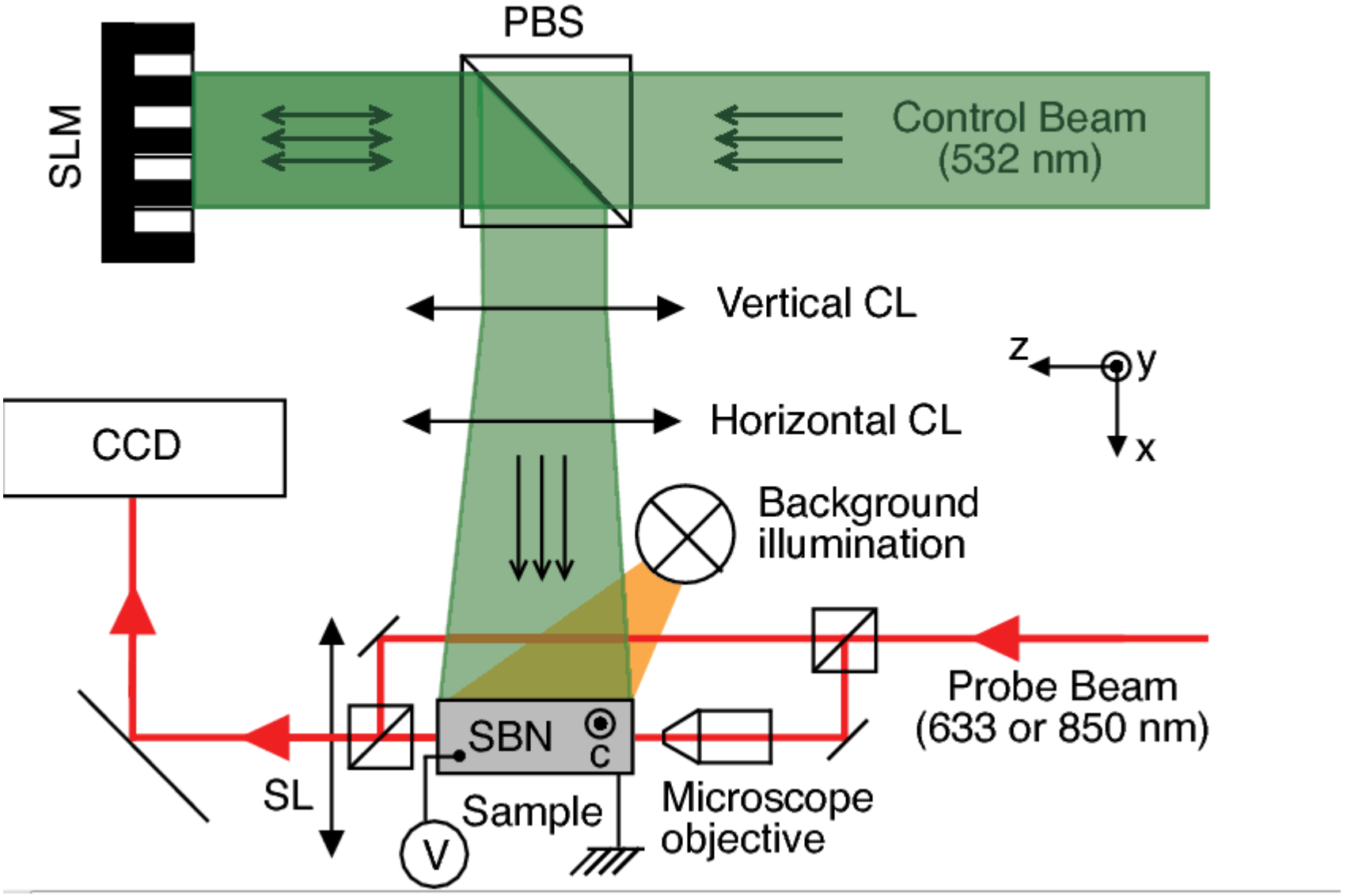
When N=5 (odd numbers of waveguides in the array), but intuitive order



When N=5 (odd numbers of waveguides in the array) and contraintuitive order



$$DarkState\left(z = L\right) = (0, 0, -1, 0, 1, 0, -1)^T$$


PBS
SLM
Control Beam
(532 nm)
Vertical CL
Horizontal CL
z
y
x
CCD
Background
illumination
Probe Beam
(633 or 850 nm)
SBN
c
SL
Sample
V
Microscope
objective

STIRAP analogy by Longhi:

Third-harmonic is generated by direct conversion without the intermediate generation of the second-harmonic field.

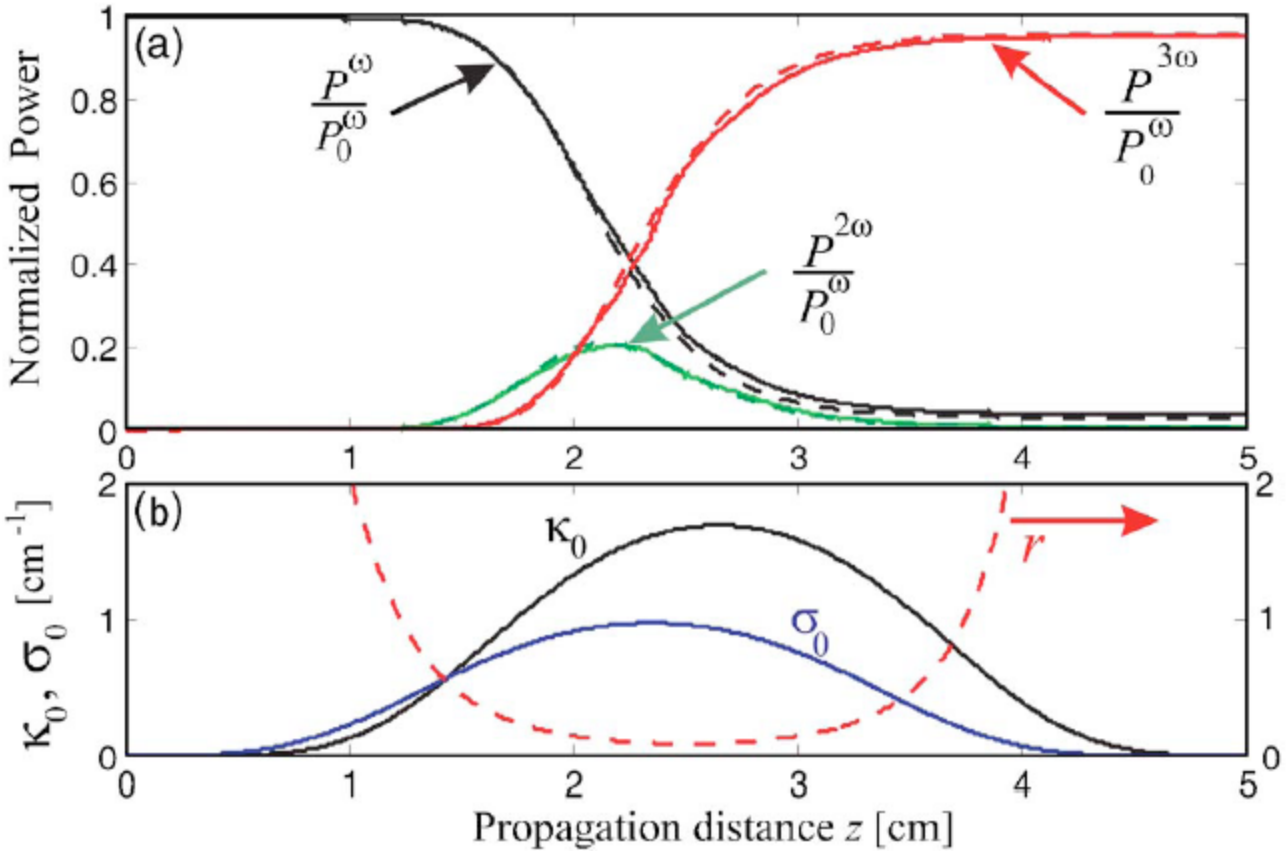
second-harmonic generation ($\omega+\omega=2\omega$)+ sum-frequency generation ($\omega+2\omega=3\omega$)

$$i\frac{dA_{\omega}}{dz}=kA_{\omega}^{*}A_{2\omega}+\sigma A_{2\omega}^{*}A_{3\omega}$$

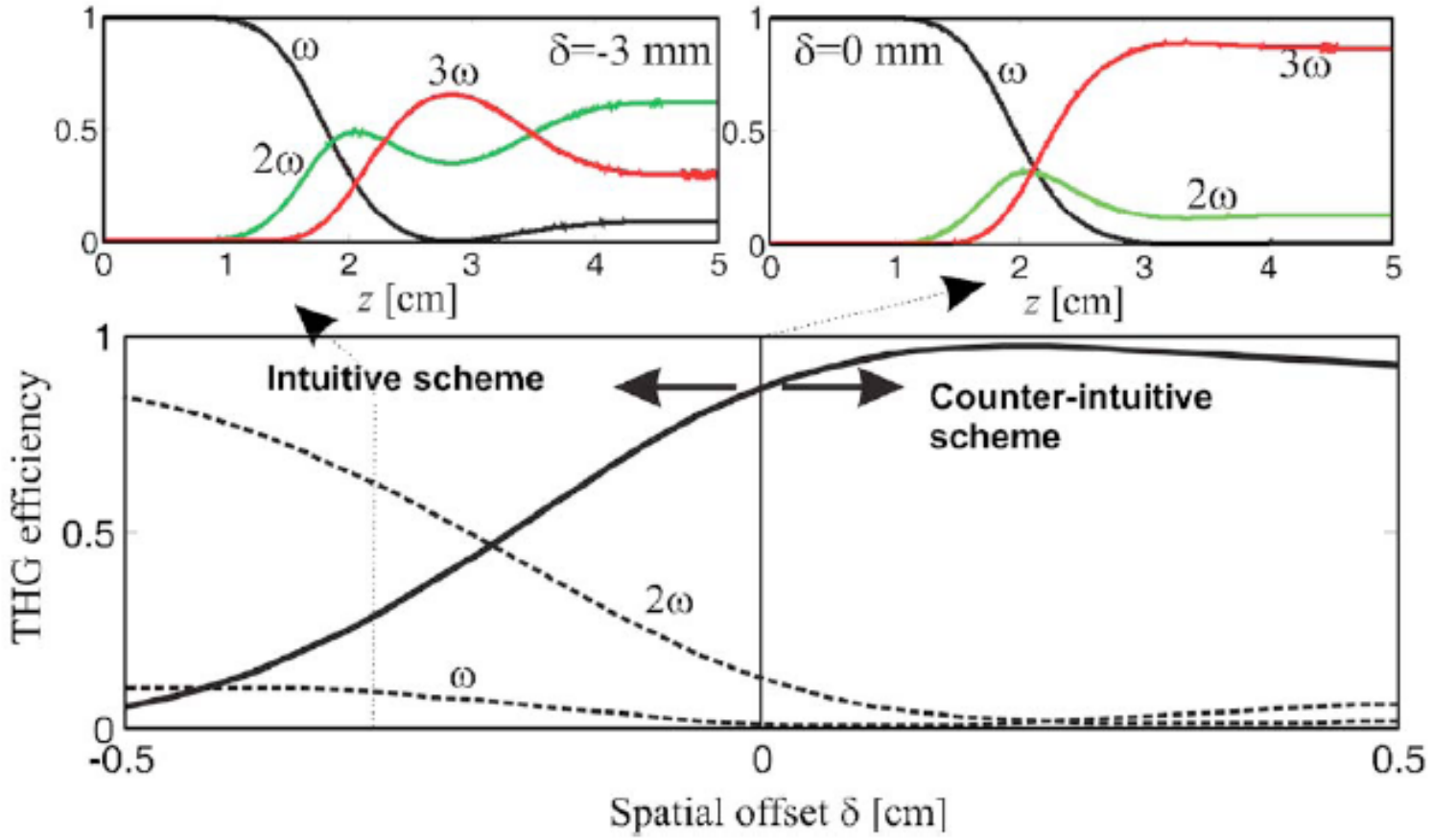
$$i\frac{dA_{2\omega}}{dz}=kA_{\omega}^{2}+2\sigma A_{\omega}^{*}A_{3\omega}$$

$$i\frac{dA_{3\omega}}{dz}=3\sigma A_{\omega}A_{2\omega}$$

S. Longhi, Opt. Lett. **32**, 1791 2007.



S. Longhi, Opt. Lett. **32**, 1791 2007.



S. Longhi, Opt. Lett. **32**, 1791 2007.

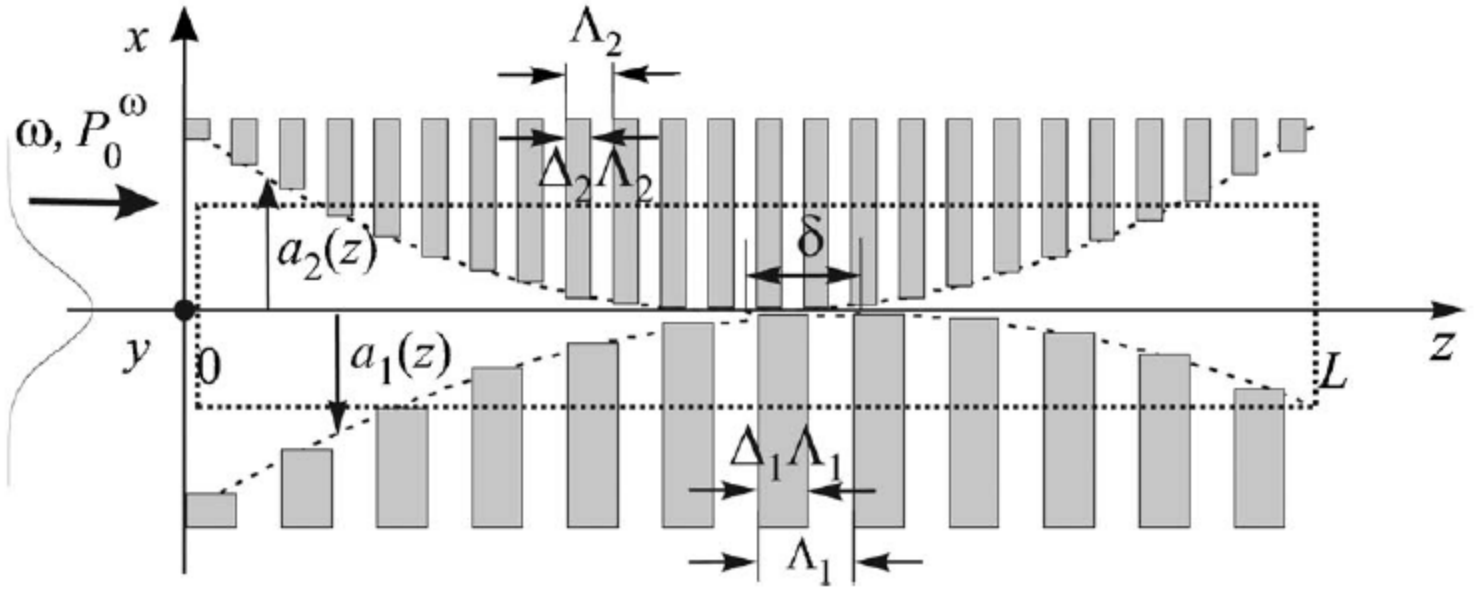
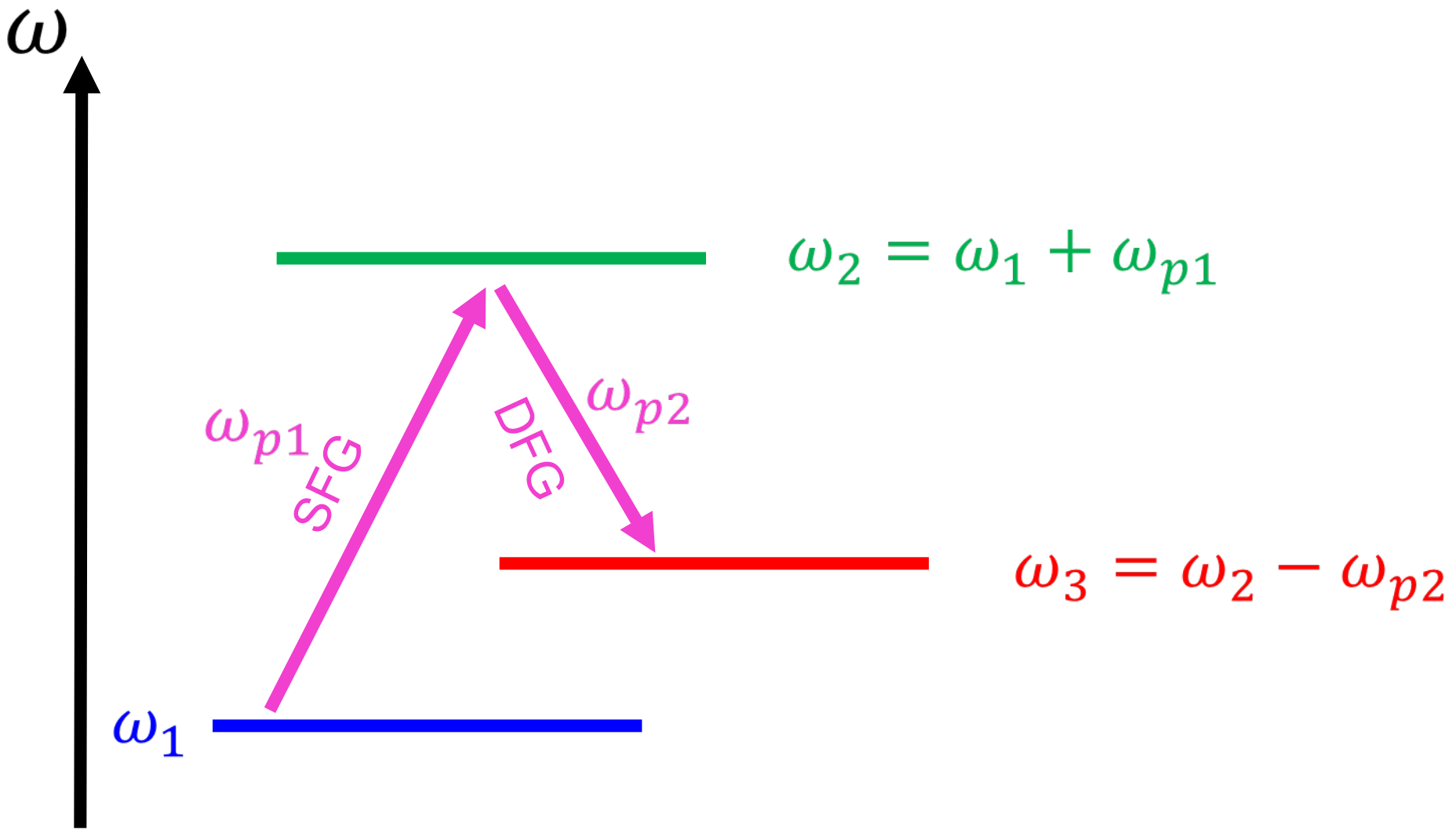
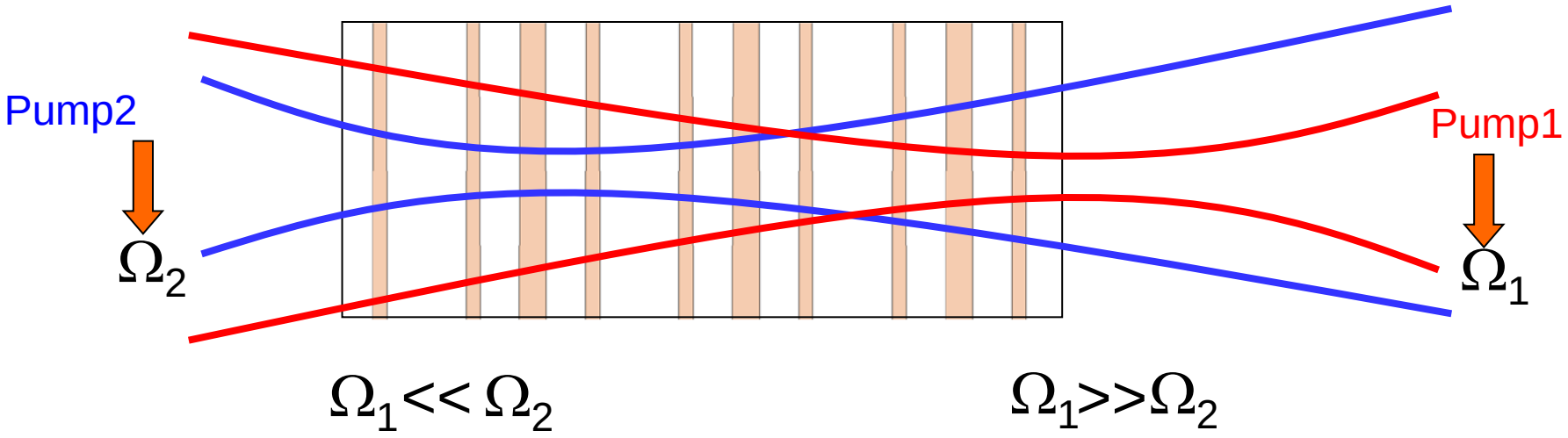


Fig. 1. Schematic of a nonlinear waveguide with a QPM grating design for adiabatic THG. The transverse patterning of the grating, with domain reversal at the shaded regions corresponding to $d(x,y,z)=-1$, provides mode-overlap control and tailoring of the coupling terms $\sigma_0(z)$ (upper grating) and $\kappa_0(z)$ (lower grating). The dotted rectangular region corresponds to the channel waveguide.

Three Wave Mixing Processes can be Cascaded

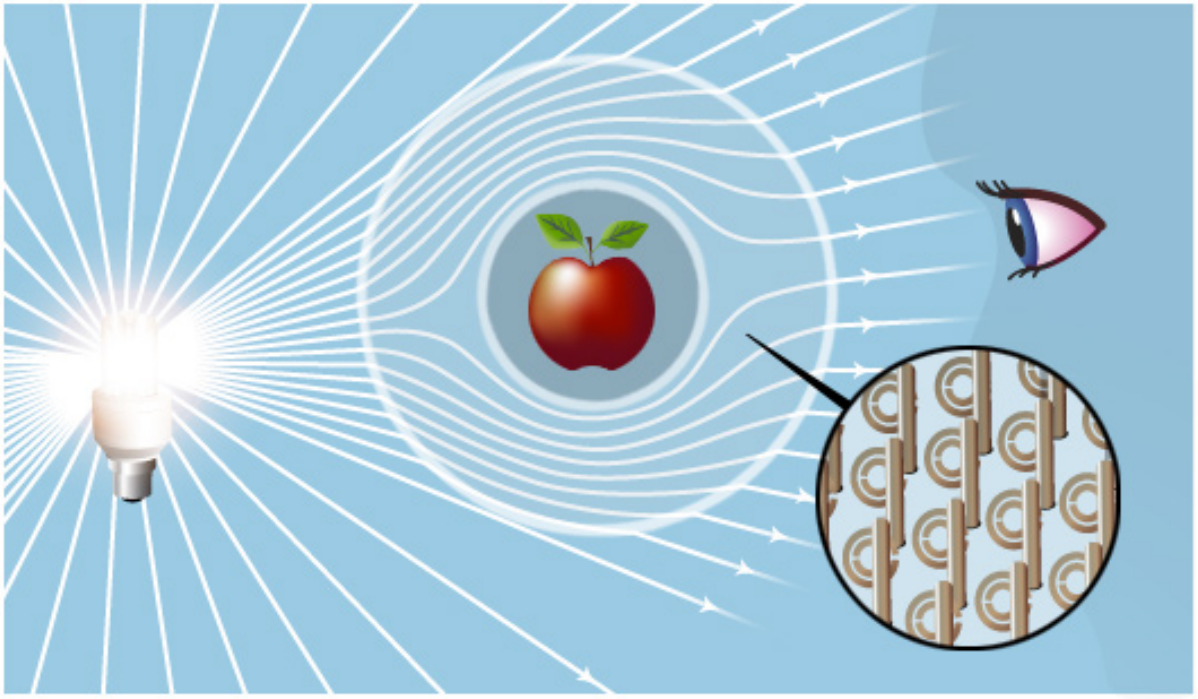


Coupling Strengths Control via Pump Focusing can be Combined with a Less-Challenging Modulation Pattern for Quasi Phase Matching

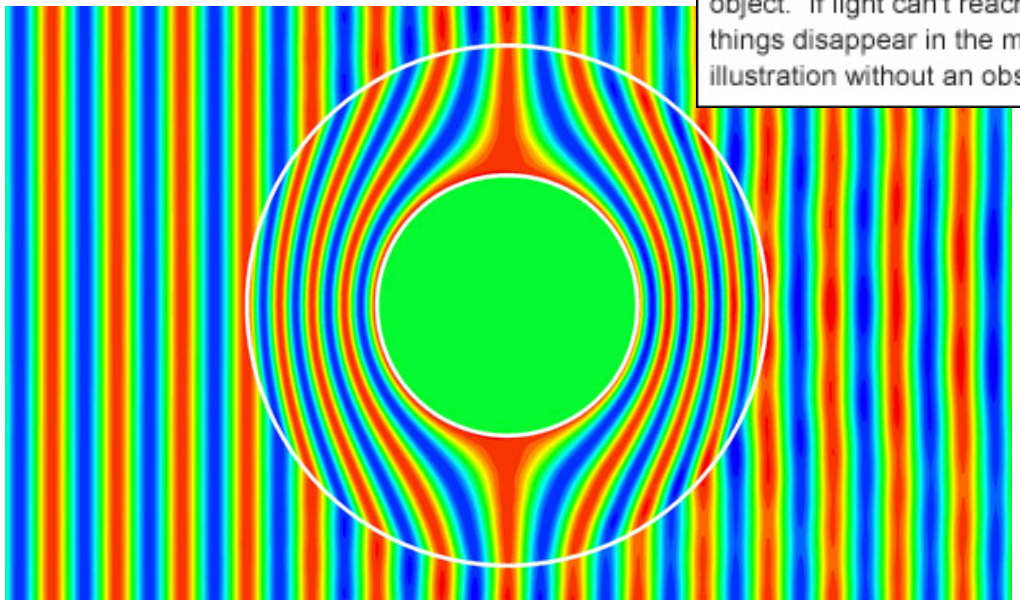


G. Porat and A. Arie, J. Opt. Soc. Am. B 29, 2901 (2012).

...now you don't !

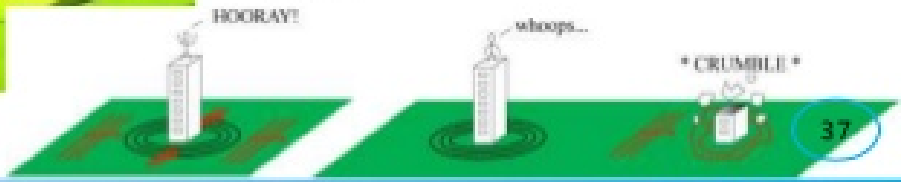


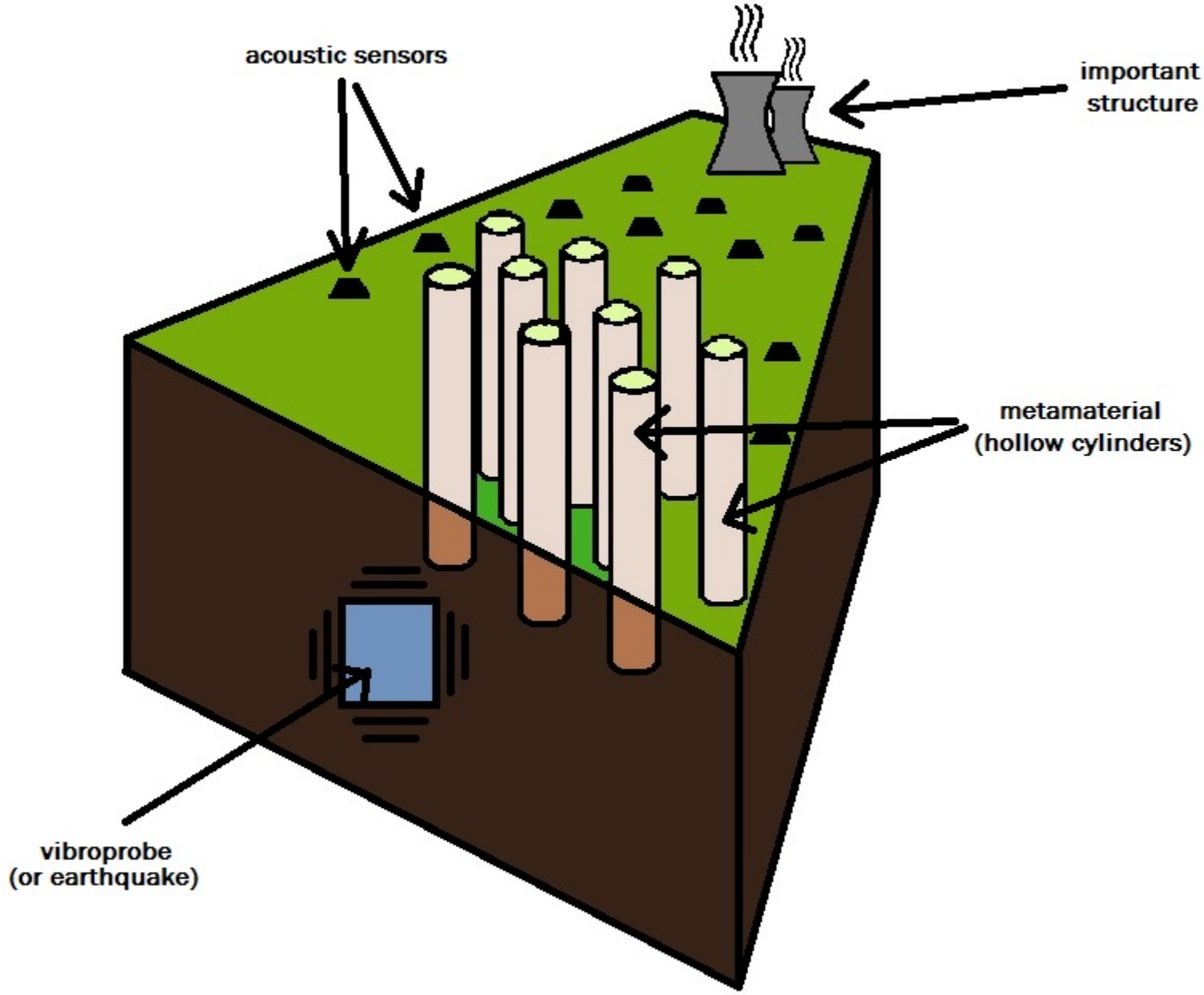
Thanks to tiny, carefully placed metal rods and rings, metamaterials can bend light around an object. If light can't reach it, we can't see it. To date scientists have only been able to make things disappear in the microwave range of frequencies, but it wouldn't be a physics illustration without an observer or an apple.

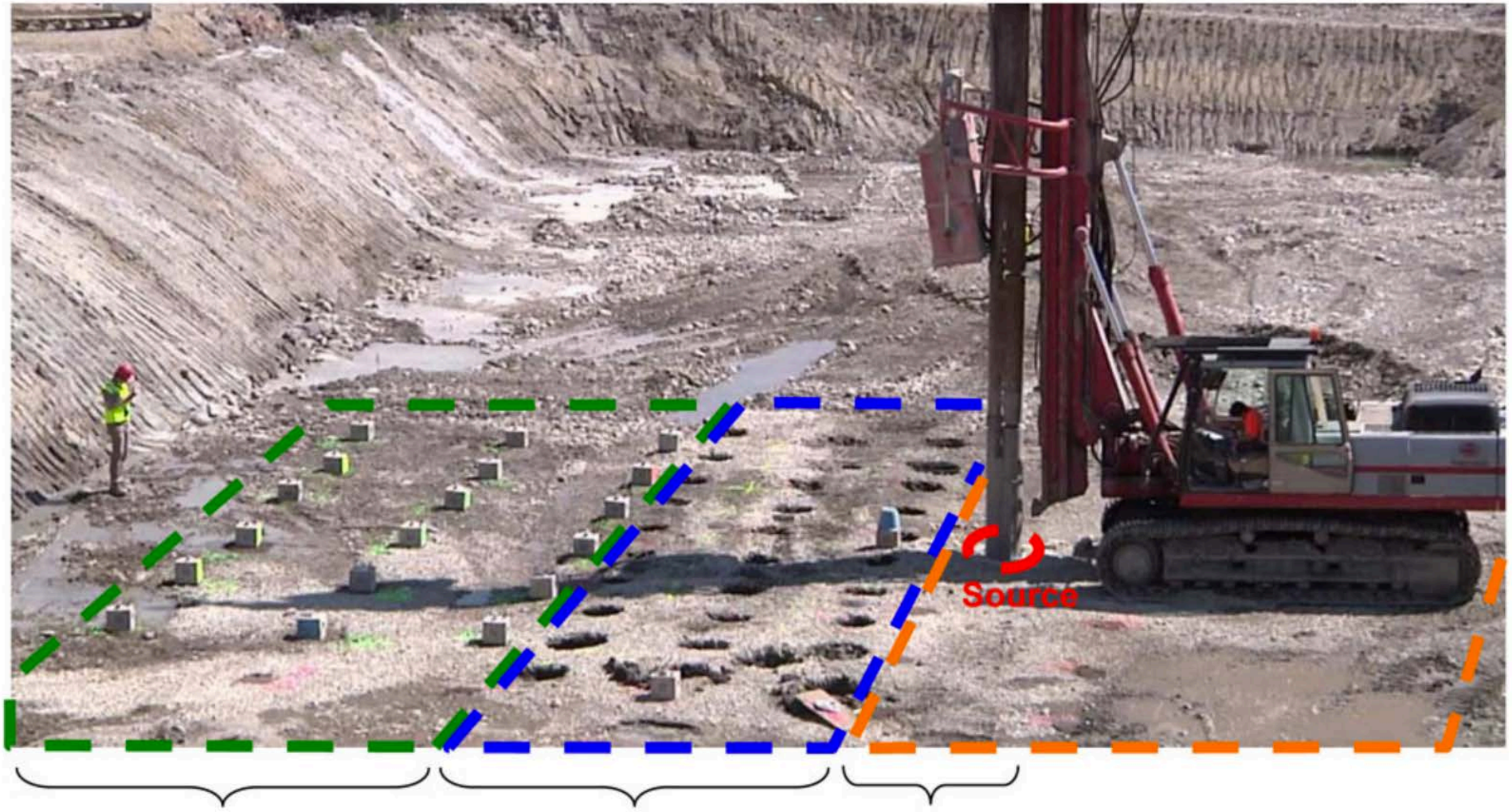


Seismic Invisibility Cloak

- Concentric plastic rings are installed underground in the top if three feet if soil.
- Harder layers are further from the building. Each time the wave hits a softer layer it is deflected off its initial path.
- Once past the building the wave can continue on their path.




acoustic sensors
important structure
metamaterial (hollow cylinders)
vibroprobe (or earthquake)



Sensitive three components velocimeters (green grid)

Five meters deep 320 mm holes

Source :
- Frequency : 50 Hz
- Horizontal displacement : 14 mm