

Stimulated Raman adiabatic passage analogues in classical physics

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Abstract

Stimulated Raman adiabatic passage (STIRAP) is a well-established technique for producing coherent population transfer in a three-state quantum system. We here exploit the resemblance between the Schrödinger equation for such a quantum system and the Newton equation of motion for a classical system undergoing torque to discuss several classical analogues of STIRAP, notably the motion of a moving charged particle subject to the Lorentz force of a quasistatic magnetic field, the orientation of a magnetic moment in a slowly varying magnetic field and the Coriolis effect. Like STIRAP, these phenomena occur for *counterintuitive* motion of the torque and are robustly insensitive to small changes in the interaction properties.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The production of complete population transfer between the quantum states of a three-state chain by means of stimulated Raman adiabatic passage (STIRAP) has become one of the staples of contemporary quantum-state manipulation [1–4]. In this technique, a pair of temporally delayed laser pulses P and S , of frequencies ω_P and ω_S respectively, act to alter the state vector $\Psi(t)$ in a three-dimensional Hilbert space from initial alignment with state ψ_1 to final alignment with state ψ_3 . The P field links state ψ_1 , of energy E_1 , to the intermediate state ψ_2 , of energy E_2 , and the S field links this to the final state ψ_3 , of energy E_3 . By applying the S field prior to the P field (so-called *counterintuitive* ordering), and maintaining two-photon resonance along with adiabatic evolution, the population transfer occurs without state ψ_2 acquiring, even temporarily, any population.

In many classical systems the equations of motion comprise three coupled equations which can be cast into the form of a torque equation, i.e. an equation of motion in which the force acting on a vector is always at right angles to the vector. The behaviour of a gyroscope acted on by gravity is a familiar example. Such a torque equation occurs in quantum optics as the well-known Bloch-vector representation of the behaviour of a coherently driven two-state quantum system

[5, 6]. It is less well known that a torque equation applies to the three-state system of STIRAP [7].

In this paper we exploit the occurrence of a torque equation in these several different areas of physics—quantum mechanics, classical mechanics and classical electrodynamics—to discuss ways in which classical motion can be altered adiabatically using two sequential but overlapping pulsed interactions. In particular, we will note the analogy with the three Cartesian coordinates of a moving charge in a magnetic field, as described by the Lorentz force [8], the orientation coordinates of a magnetic moment in a magnetic field, as described by the Landau–Lifshitz–Gilbert equation [9, 10] and the velocity change of a moving particle due to change in Coriolis force.

We note that several authors have discussed analogies between three-state quantum systems and classical systems. These similarities include an analogue with the motion of a classical pendulum [11] and an analogue with electromagnetically induced transparency [12].

This paper is organized as follows. In section 2 we present the basic mathematics of STIRAP, and cast the equations into the form of a torque equation. In section 3 we present the equations of motion for a charged particle subject to a Lorentz force, with specialization that makes these identical to the STIRAP torque equation. Specifically, we consider a particle

that initially moves in the z direction, acted upon by a sequence of two magnetic-field pulses. The first of these has only a z component (thereby producing no change in the motion), while the second has only an x component. The resulting particle motion is in the x direction; there is never any component in the y direction, despite the presence of the x -directed magnetic field. In sections 4 and 5 we discuss two more examples of classical physics where STIRAP-like processes can occur. These include the reorientation of magnetization and the Coriolis effect. Section 6 presents a summary.

2. STIRAP

The basic equation of motion governing STIRAP is the time-dependent Schrödinger equation, in the rotating-wave approximation (RWA). Expressed in vector form, this reads

$$i\hbar \frac{d}{dt} \mathbf{c}(t) = \mathbf{H}(t)\mathbf{c}(t), \quad (1)$$

where $\mathbf{c}(t)$ is a column vector of probability amplitudes $c_n(t)$ ($n = 1, 2, 3$) and $\mathbf{H}(t)$ is the RWA Hamiltonian matrix [13–15]. In the example of STIRAP there are three basic quantum states— ψ_1, ψ_2 and ψ_3 —linked as a chain 1–2–3, and $\mathbf{H}(t)$ is a 3×3 matrix,

$$\mathbf{H}(t) = \frac{\hbar}{2} \begin{bmatrix} 0 & \Omega_P(t) & 0 \\ \Omega_P(t) & 0 & \Omega_S(t) \\ 0 & \Omega_S(t) & 0 \end{bmatrix}. \quad (2)$$

Here the two slowly varying Rabi frequencies $\Omega_P(t)$ and $\Omega_S(t)$ parameterize the strengths of the pulsed P - and S -field interactions; they are proportional to dipole transition moments d_{ij} and to electric-field amplitudes $\mathcal{E}_k(t)$,

$$\hbar\Omega_P = -d_{12}\mathcal{E}_P(t), \quad \hbar\Omega_S = -d_{23}\mathcal{E}_S(t), \quad (3)$$

and hence they vary as the square root of pulse intensities. We take these to be real-valued functions of time. We have here assumed not only two-photon resonances $\hbar|\omega_P - \omega_S| = |E_3 - E_1|$, as required for STIRAP, but also single-photon resonances: $|E_2 - E_1| = \hbar\omega_P$, $|E_2 - E_3| = \hbar\omega_S$.

The quantum evolution associated with STIRAP is most easily understood with the use of adiabatic states, i.e. the three instantaneous eigenstates $\Phi_k(t)$ of the RWA Hamiltonian, defined as $\mathbf{H}(t)\Phi_k(t) = \hbar\varepsilon_k(t)\Phi_k(t)$,

$$\Phi_+(t) = [\psi_1 \sin \vartheta(t) + \psi_2 + \psi_3 \cos \vartheta(t)]/\sqrt{2}, \quad (4a)$$

$$\Phi_0(t) = \psi_1 \cos \vartheta(t) - \psi_3 \sin \vartheta(t), \quad (4b)$$

$$\Phi_-(t) = [\psi_1 \sin \vartheta(t) - \psi_2 + \psi_3 \cos \vartheta(t)]/\sqrt{2}. \quad (4c)$$

Here the time-dependent mixing angle $\vartheta(t)$ is defined as the ratio of interaction strengths,

$$\tan \vartheta(t) = \frac{\Omega_P(t)}{\Omega_S(t)}. \quad (5)$$

The adiabatic state $\Phi_0(t)$ is particularly noteworthy: it has a null eigenvalue and it has no component of state ψ_2 —it therefore does not lead to fluorescence from that state; it is a *dark* state [1–4, 16–19]. The construction of all three adiabatic states varies as the pulse sequence alters the mixing angle, but $\Phi_0(t)$ remains at all times within a two-dimensional Hilbert subspace. It is this property that we exploit for our classical analogies.

The two alternative pulse orderings, $S - P$ and $P - S$, lead to qualitatively different results.

2.1. Counterintuitive pulse order: STIRAP

The STIRAP mechanism relies on maintaining a continuing alignment of the state vector $\Psi(t)$ with the dark state $\Phi_0(t)$, and having this adiabatic state initially aligned with state ψ_1 . To have this initial alignment it is necessary that the S field act first. Because this field has no interaction linkage with the initially populated state ψ_1 it does not directly produce population transfer. Thus the S -before- P sequence is termed *counterintuitive*. With this ordering the mixing angle $\vartheta(t)$ and the adiabatic state $\Phi_0(t)$ have the behaviour

$$0 \xleftarrow{-\infty \leftarrow t} \vartheta(t) \xrightarrow{t \leftarrow +\infty} \frac{\pi}{2}, \quad (6a)$$

$$\psi_1 \xleftarrow{-\infty \leftarrow t} \Phi_0(t) \xrightarrow{t \leftarrow +\infty} -\psi_3. \quad (6b)$$

That is, the $S - P$ pulse sequence rotates the adiabatic state $\Phi_0(t)$ from alignment with the initial state ψ_1 to alignment with the target state ψ_3 . If the motion is adiabatic, then the state vector $\Psi(t)$ follows this same Hilbert-space rotation. The result is complete population transfer. Moreover, the motion of the state vector remains entirely in a two-dimensional subspace of the three-dimensional Hilbert space. This restriction allows a simple description of the dynamics as a torque equation. The condition for adiabatic evolution amounts to the requirement of large temporal pulse areas, $A_k = \int_{-\infty}^{\infty} \Omega_k(t) dt$ ($k = P, S$) [2–4],

$$A_P \gg 1, \quad A_S \gg 1. \quad (7)$$

2.2. Intuitive pulse order: oscillations

The *intuitive* sequence $P - S$ produces populations that display Rabi-like oscillations [20]. This behaviour is readily understood by viewing the construction of the adiabatic states for very early and very late times. Let the state vector coincide with state ψ_1 at time $t \rightarrow -\infty$, when the S field is absent. Then this initial state has the construction (cf equations (4))

$$\Psi(-\infty) = \psi_1 = [\Phi_+(-\infty) + \Phi_-(-\infty)]/\sqrt{2}. \quad (8)$$

The two separate evolution paths lead to oscillations of the populations [20],

$$P_1 = 0, \quad P_2 = \sin^2 \frac{1}{2}A, \quad P_3 = \cos^2 \frac{1}{2}A, \quad (9)$$

where A is the rms pulse area,

$$A = \int_{-\infty}^{\infty} \sqrt{\Omega_P^2(t) + \Omega_S^2(t)} dt. \quad (10)$$

Thus, only for certain values of the pulse area (generalized π -pulses), it is possible to obtain complete population transfer from state ψ_1 to state ψ_3 with a resonant $P - S$ pulse sequence.

2.3. The STIRAP torque equation

The three-state Schrödinger equation driven by the Hamiltonian (2) has, with a redefinition of variables,

$$R_1(t) = -c_3(t), \quad R_2(t) = -ic_2(t), \quad R_3(t) = c_1(t), \quad (11)$$

the form

$$\frac{d}{dt} \begin{bmatrix} R_1(t) \\ R_2(t) \\ R_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -\Omega_S(t) & 0 \\ \Omega_S(t) & 0 & -\Omega_P(t) \\ 0 & \Omega_P(t) & 0 \end{bmatrix} \begin{bmatrix} R_1(t) \\ R_2(t) \\ R_3(t) \end{bmatrix} \quad (12)$$

The symmetry of these coupled equations allows us to write them as a torque equation [7],

$$\frac{d}{dt} \mathbf{R}(t) = \Omega(t) \times \mathbf{R}(t), \quad (13)$$

where we have introduced the angular velocity vector

$$\Omega(t) = [-\Omega_P(t), 0, \Omega_S(t)]^T. \quad (14)$$

Like torque equations in classical dynamics [21], this equation says that changes in a vector, $\mathbf{R}(t)$ in this case, are produced by a force $\Omega(t) \times \mathbf{R}(t)$ that is perpendicular to the vector. Such an equation occurs in the description of two-state excitation, where it geometrizes the Bloch equation [5]. In all cases it describes an instantaneous rotation of the vector $\mathbf{R}(t)$, in the plane orthogonal to the direction of the angular velocity vector at an instantaneous rate $|\Omega(t)| = \sqrt{\Omega_P(t)^2 + \Omega_S(t)^2}$.

The dark state (4b) written for the vector $\mathbf{R}(t)$ gives the *dark superposition*

$$R_0(t) = \frac{\Omega_P(t)R_1(t) + \Omega_S(t)R_3(t)}{|\Omega(t)|}. \quad (15)$$

The STIRAP evolution, regarded as the solution to the torque equation (13), comprises the following motion. Initially $\mathbf{R}(t)$ points along the z -axis and $\Omega(t)$ points along this same axis. Because these two vectors are collinear the vector $\mathbf{R}(t)$ does not move. The introduction of the P field rotates $\Omega(t)$ in the xz -plane towards the x -axis. Because this motion is adiabatic, it causes the vector $\mathbf{R}(t)$ to follow. In the end, both $\mathbf{R}(t)$ and $\Omega(t)$, remaining collinear, are aligned along the x -axis.

We note that the conservation of the length of the vector $\mathbf{R}(t)$, which follows from the torque equation (13), is equivalent to the conservation of probability ensuing from equation (1).

The following sections will note several classical systems that are governed by a torque equation, and will discuss the analogues of STIRAP motion for these systems.

3. STIRAP in Lorentz force

A charged particle moving in a magnetic field is affected by a force that is perpendicular to both the velocity and the magnetic field. Let the particle have a mass m , a charge q and a velocity $\mathbf{v}(t) = [v_x(t), v_y(t), v_z(t)]^T$. Let the magnetic field be restricted to components in the xz -plane, $\mathbf{B}(t) = [-B_x(t), 0, B_z(t)]^T$. The Lorentz force acting on the particle is $q\mathbf{v} \times \mathbf{B}$. The Newton's equation of motion therefore appears as a torque equation for the particle velocity,

$$m \frac{d}{dt} \mathbf{v} = -q\mathbf{B} \times \mathbf{v}. \quad (16)$$

Drawing an analogy to STIRAP, we write down a *dark-velocity* superposition $V_0(t)$ of the velocity components $v_x(t)$ and $v_z(t)$,

$$V_0(t) = \frac{B_x(t)v_x(t) + B_z(t)v_z(t)}{|\mathbf{B}(t)|}. \quad (17)$$

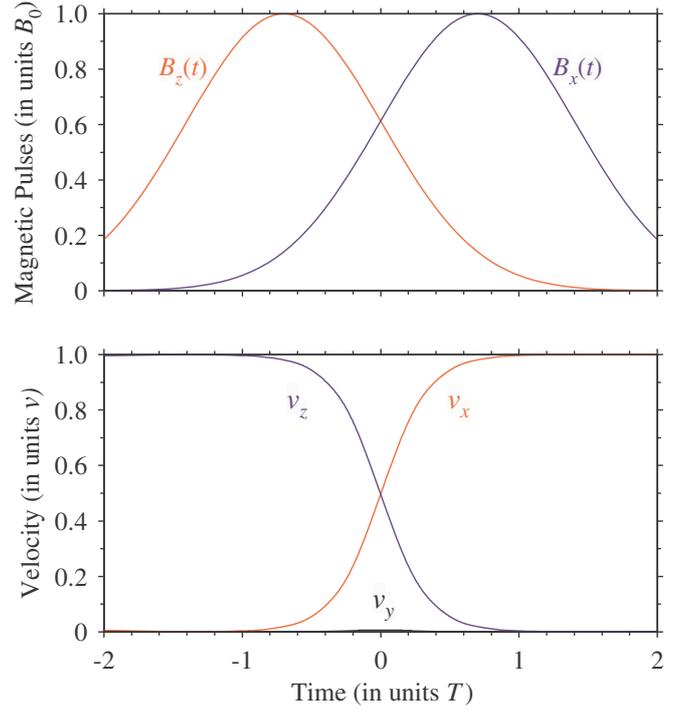


Figure 1. Evolution of the magnetic vector (top frame) and the velocity vector of the moving charged particle (bottom frame). For *counterintuitive* order of pulses (top frame) the initial velocity changes from alignment with z -axis to alignment with x -axis without having any components on the y -axis. We have assumed Gaussian magnetic pulse shapes, $B_x(t) = B_0 e^{-(t+\tau/2)^2/T^2}$, $B_z(t) = B_0 e^{-(t-\tau/2)^2/T^2}$, with $B_0 = 20T^{-1}$ and $\tau = -1.2T$.

When $B_z(t)$ precedes $B_x(t)$ then the dark velocity superposition $V_0(t)$ has the asymptotic values

$$v_z(-\infty) \xleftarrow{-\infty \leftarrow t} V_0(t) \xrightarrow{t \rightarrow +\infty} v_x(\infty). \quad (18)$$

Thus if initially the particle travels along the z direction, $\mathbf{v}(-\infty) = [0, 0, v]^T$, we can direct the velocity to the x direction by applying first a field in the z direction and then slowly rotating this to the x direction as shown in figure 1. The initial field, being in the direction of motion, has no effect; in this sense the pulse sequence is *counterintuitive*.

If the initial magnetic field is in the x direction (*intuitive* pulse order), then the charged particle, which travels initially along the z -axis, will be subjected to a Lorentz force and will begin a Larmor precession in the yz -plane, with radius $r_L = mv/qB_0$. Then, as the \mathbf{B} -field switches from x to z direction, the particle precession will turn into the xy -plane. The final velocity of the particle depends on the value of the accumulated precession angle A (cf equations (9) and (11)): $\mathbf{v}(+\infty) = v[\cos^2(A/2), \sin^2(A/2), 0]^T$.

These features are demonstrated in figure 2, where the velocity components of the charged particle are plotted versus the delay between the magnetic pulses $B_x(t)$ and $B_z(t)$. A flat plateau of high values of v_x is observed for negative delays (counterintuitive pulse order), whereas oscillations between v_x and v_y occur for positive delays. Note that the final value of v_z does not depend on the sign of the delay τ [22].

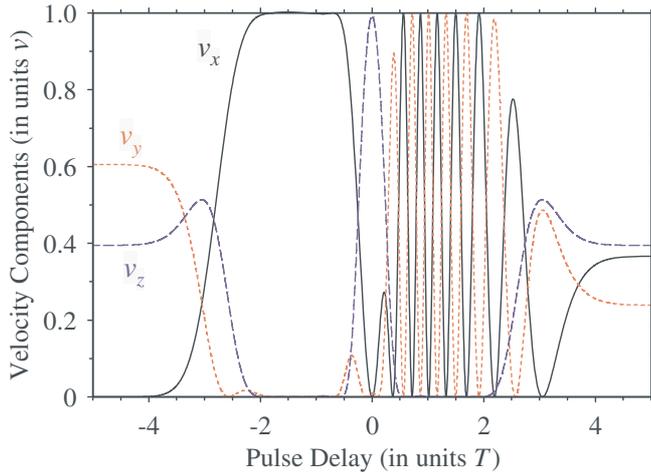


Figure 2. Numerically calculated velocity components $v_x(t)$ (dashed line), $v_y(t)$ (dotted line) and $v_z(t)$ (solid line) as a function of the pulse delay. We have assumed Gaussian magnetic pulse shapes, $B_x(t) = B_0 e^{-(t+\tau/2)^2/T^2}$, $B_z(t) = B_0 e^{-(t-\tau/2)^2/T^2}$ with $B_0 = 40T^{-1}$.

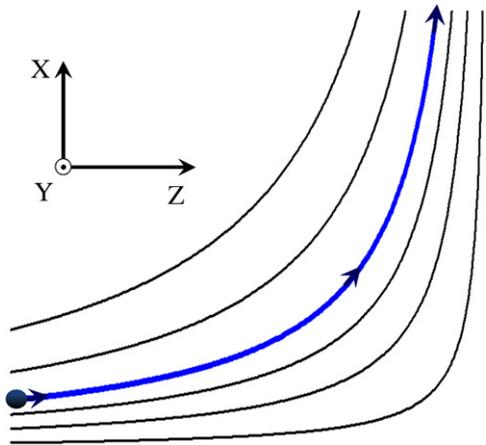


Figure 3. Spatial geometry of the magnetic field lines that produces the STIRAP scheme for the Lorentz force. The thick line illustrates the particle trajectory in the adiabatic limit.

The equation of motion (16) holds only for quasistatic fields. To implement the desired STIRAP analogy we can use a spatial arrangement of the magnetic field such that the components appear to the moving particle as two sequential, but overlapping magnetic fields. This spatial geometry, viewed in the reference frame of the particle allows us to write the magnetic field as time dependent without any associated electric field. Figure 3 shows an example of spatial geometry that gives the desired ordering of the magnetic field component. Note that in the adiabatic regime the particle follows a field line.

Following quantum-optical STIRAP, we conclude that the condition for adiabatic evolution in this Lorenz-STIRAP is a large value of the accumulated precession angle A (which is the pulse area in quantum-optical STIRAP). Within the above-mentioned spatial arrangement (with a characteristic length L), the adiabatic evolution condition sets an upper limit on

the charged particle velocity v , or lower limits on the peak magnetic field B_0 and the length L ,

$$mv \ll qB_0L. \quad (19)$$

Note that the quantity mv/qB_0 is the radius of the Larmor orbit r_L ; hence condition (19) implies $r_L \ll L$.

4. Magnetization

A magnetic field \mathbf{H} acts to turn a magnetic moment $\mathbf{M}(t) = [M_x(t), M_y(t), M_z(t)]^T$, with a force that is always perpendicular to $\mathbf{M}(t)$. The system dynamics is expressible again as a torque equation,

$$\frac{d}{dt}\mathbf{M}(t) = \gamma\mathbf{M}(t) \times \mathbf{H}(t), \quad (20)$$

where γ is the gyromagnetic ratio. This is the homogeneous Bloch equation for magnetization with infinite relaxation times [6]. It is also known in the literature as the undamped case of the Landau–Lifshitz–Gilbert equation [9, 10]. The *dark* superposition for the magnetic moment reads

$$M_0(t) = \frac{H_x(t)M_x(t) + H_z(t)M_z(t)}{|\mathbf{H}(t)|}. \quad (21)$$

When $\mathbf{H}(t) = [-H_x(t), 0, H_z(t)]$, and the magnetic component $H_z(t)$ precedes the magnetic component $H_x(t)$, the dark magnetic moment $M_0(t)$ has the asymptotics

$$M_z(-\infty) \xrightarrow{t \rightarrow -\infty} M_0(t) \xrightarrow{t \rightarrow +\infty} M_x(+\infty). \quad (22)$$

Thus if we start with the initial magnetic moment pointed in the z direction, $\mathbf{M}(-\infty) = [0, 0, M]^T$, we can change the direction of the magnetization from the z -axis to the x -axis by applying first a magnetic pulse $H_z(t)$ and then a magnetic pulse $H_x(t)$ (counterintuitive order), while maintaining adiabatic evolution. Because the adiabatic passage is robust, this procedure is robust: it depends only weakly on the overlap of the two magnetic components and the peak values of $H_x(t)$ and $H_z(t)$.

5. The Coriolis effect

In classical mechanics, the Coriolis effect is the apparent deflection of a moving object when it is viewed in a rotating reference frame. The vector formula for the magnitude and direction of the Coriolis acceleration is

$$\frac{d}{dt}\mathbf{v} = 2\mathbf{v} \times \boldsymbol{\omega}, \quad (23)$$

where $\mathbf{v}(t) = [v_x(t), v_y(t), v_z(t)]^T$ is the velocity of the particle in the rotating system and $\boldsymbol{\omega}(t) = [\omega_x(t), \omega_y(t), \omega_z(t)]^T$ is the angular velocity vector of the rotating frame. This equation has the same vector form as equation (13) and therefore, a STIRAP-like process may occur if the angular velocity vector of the rotating frame changes appropriately.

Consider the following example: a sphere rotates around the z -axis with an angular velocity $\boldsymbol{\omega} = [0, 0, \omega]^T$ and a point particle moves along the same axis (figure 4, thick blue arrow). Then the angular velocity of the sphere $\boldsymbol{\omega}$ rotates adiabatically from z - to x -axis; the particle velocity

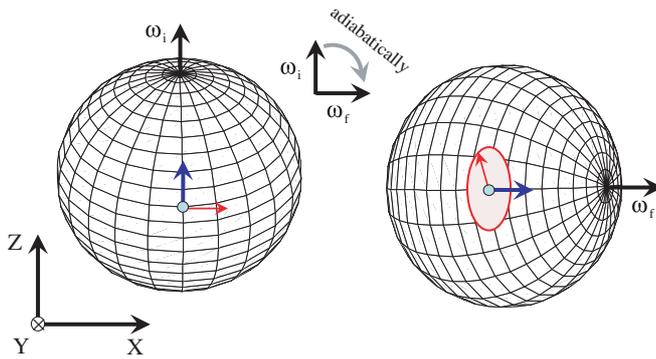


Figure 4. Example of STIRAP in the Coriolis effect. As the axis of rotation of a sphere rotates adiabatically from the z - to the x -axis, a point particle, moving on the surface of the sphere initially along the z -axis, changes its velocity from the z - to the x -axis as well (thick blue arrow). However, a point particle, moving initially along the x -axis, ends up with a velocity in the yz -plane (thin red arrow); its final direction in this plane is determined by a ‘pulse area’, which is related to the time of rotation and the magnitude of the angular velocity ω .

$\mathbf{v}(t)$ follows adiabatically this rotation and ends up aligned with the x -axis. In contrast, if the initial velocity of the particle is along the x -axis (thin red arrow), then it will end up with its velocity in the yz -plane. The precise direction is determined by the ‘temporal pulse area A ’, according to equation (9), as $\mathbf{v}(+\infty) = v[\cos^2(A/2), \sin^2(A/2), 0]^T$. The adiabatic condition in this case may look surprising: it requires that the product of the angular velocity ω and the time of its rotation T be large: $\omega T \gg 1$. In other words, a ‘slow’ change in the direction of the angular velocity ω requires a large magnitude of ω .

6. Conclusions

We have presented several examples of well-known dynamical problems in classical physics, which demonstrate that the elegant and powerful technique of STIRAP in quantum optics is not restricted to quantum systems. The application of STIRAP to these problems, which appears experimentally easily feasible, is intriguing and offers a potentially useful and efficient control technique for classical dynamics.

The first factor that enables this analogy is the equivalence of the Schrödinger equation for a fully resonant three-state quantum system, wherein the quantum-optical STIRAP operates, to the optical Bloch equation for a two-state quantum system. The second factor is the Feynmann–Vernon–Hellwarth vector form of the Bloch equation, which has the form of a torque equation, i.e. the force on a vector is perpendicular to the vector.

In the Lorentz force case, the variables for the STIRAP analogy are velocity components. The STIRAP procedure changes the direction of the velocity from the z -axis to the x -axis with never a component along the y -axis. The procedure has the same efficiency and robustness as STIRAP. The described technique for a Lorentz force is not only a curious and intriguing example of the adiabatic passage, but

it also has the potential to be a useful, efficient and robust technique for magnetic shielding, magnetic lenses or speed selection of charged particles.

Applied to the equation of motion of a magnetic moment in a magnetic field, the analogy of STIRAP offers a robust mechanism for changing the orientation of a magnetic moment. STIRAP-like processes can also be designed in other intriguing physical situations, such as the Coriolis effect. An equation of a torque form emerges also in the description of the effect of general relativistic gravitational *frame dragging*, e.g. when a massive spinning neutral particle is placed at the centre of a unidirectional ring laser [23]. Then the linearized Einstein field equations in the weak-field and slow-motion approximation lead to an equation for the spin of the same form as equation (13).

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